ASPG
American Scientific Publishing Group

INTERNATIONAL JOURNAL OF NEUTROSOPHIC SCIENCE

Volume 11, 2020

Editor(s) in Chief: Broumi Said & Florentin Smarandache

ISSN(ONLINE): 2690-6805
ISSN(PRINT) : 22692-6148

Table of Content

International Journal of Neutrosophic Science (IJNS)

International Journal of Neutrosophic Science (IJNS) ABSTRACTED/INDEXED IN

Google Scholar

Index Copernicus (ICI World of Journals)

BASE Search

Microsoft Academic

Advanced Science Index (ASI)

EuroPub

WorldCat

Editorial Board

S. A. Edalatpanah, Department of Applied Mathematics, Ayandegan Institute of Higher Education, Tonekabon, Iran(saedalatpanah@gmail.com).

Liu Chun Feng, Shenyang Aerospace University, China (liuchunfang1112@163.com).

Wadei faris Mohammed Al-omeri, Department of Mathematics, Faculty of Science, Al-Balqa Applied University, Salt 19117, Jordan (wadeimoon1@hotmail.com, wadeialomeri@bau.edu.jo).

Abhijit Saha, Department of Mathematics, Techno College of Engineering Agartala Maheshkhola-799004, Tripura, India (abhijit84.math@gmail.com).

D.Nagarajan, Department of Mathematics, Hindustan Institute of Technology & Science, India (dnagarajan@hindustanuniv.ac.in).

Prem Kumar Singh, Amity Institute of Information Technology, Amity University-Sector 125 Noida-201313, Uttar Pradesh-India (premsingh.csjm@gmail.com).

Avishek Chakraborty, Department of Basic Science, University/College- Narula Institute of Technology Under MAKAUT, India (tirtha.avishek93@gmail.com).

Arindam Dey, Department of Computer Science and Engineering, Saroj Mohan Institute of Technology, Hooghly 712512, West Bengal, India (arindam84nit@gmail.com).

Muhammad Gulistan, Department of Mathematics & Statistics, Hazara University Mansehra, Khyber Pakhtunkhwa, Pakistan(**gulistanmath@hu.edu.pk**).

Mohsin Khalid, The University of Lahore, Pakistan(**mk4605107@gmail.com**).

Le Hoang Son, PhD, Vietnam National University, Hanoi, Vietnam, (sonlh@vnu.edu.vn).

Kishore Kumar P.K, Department of Information Technology, Al Musanna College of Technology, Sultanate of Oman (**kishore2982@gmail.com**).

Mohamed Talea, Laboratory of Information processing, Faculty of Science Ben M'Sik, Morocco (taleamohamed@yahoo.fr)

Assia Bakali, Ecole Royale Navale, Casablanca, Morocco (assiabakali@yahoo.fr).

Tahir Mahmood, Department of Mathematics and Statistics, International Islamic University Islamabad, Pakistan (**tahirbakhat@iiu.edu.pk**).

Faruk KARAASLAN, Çankırı Karatekin University, Faculty of Sciences, Department of Mathematics, 18100, Çankırı, TURKEY(fkaraaslan@karatekin.edu)

Mohamed Abdel-Basset, Department of Computer Science, Zagazig University, Egypt (**analyst_mohamed@yahoo.com, analyst_mohamed@zu.edu.eg).**

Riad Khider AlHamido, Department of mathematics Faculty of Sciences - Al- Furat University, Syria (riad-hamido1983@hotmail.com).

Fahad Mohammed Alsharari, Mathematics Department, College of Science and Human Studies at Hotat Sudair, Majmaah University, Saudi Arabia (f.alsharari@mu.edu.sa).

Maikel Leyva Vázquez, Universidad Politécnica Salesiana, Ecuador (**mleyvaz@gmail.com).**

Ir. Victor Christianto, DDiv. (associate of NSIA), Satyabhakti Advanced School of Theology - Jakarta Chapter, Indonesia(**victorchristianto@gmail.com**).

Xiaohong Zhang, Shaanxi University of Science and Technology, China (zhangxiaohong@sust.edu.cn, zxhonghz@263.net).

Huda E. Khalid, Head of the Scientific Affairs and Cultural Relations, Presidency of Telafer University ,Iraq (hodaesmail@yahoo.com, dr.huda-ismael@uotelafer.edu.iq).

Ranjan Kumar, Department of Mathematics, National Institute of Technology, Adityapur, Jamshedpur, 831014, India. (**ranjank.nit52@gmail.com**).

Choonkil Park, Department of Mathematics, Hanyang University, Republic of Korea (baak@hanyang.ac.kr).

Shahzaib Ashraf, Department of Mathematic Abdul Wali Khan University, Mardan 23200, Pakistan (shahzaibashraf@awkum.edu.pk).

Madeleine Al-Tahan, Lebanese International University,Lebanon (madeline.tahan@liu.edu.lb).

Rafif alhabib, Department of mathmatical statistic, Faculty of Sciences, albaath University, Syria. (rafif.alhabib85@gmail.com)

Giorgio Nordo, MIFT - Department of Mathematical and Computer Science, Physical Sciences and Earth Sciences, Messina University, Italy,(giorgio.nordo@unime.it)

Angelo de Oliveira, Departamento Academico de Ciencia da Computacao - Universidade Federal de Rondonia, Brasil, (angelo@unir.br)

A.A.A. Agboola, Department of Mathematics, Federal University of Agriculture, Abeokuta, Nigeria,(agboolaaaa@funaab.edu.ng)

A. A. Salama, Department of Mathematics and Computer Science, Faculty of Science, Port Said University, Port Said, Egypt, (drsalama44@gmail.com)

Akbar Rezaei, Department of Mathematics, Payame Noor University, P.O..Box. 19395-3697, Tehran, IRAN, (rezaei@pnu.ac.ir)

Ahmed Hatip, Department of Mathematics, Gaziantep University, Turkey, (kollnaar5@gmail.com)

Metawee Songsaeng , Department of Mathematics, School of Science, University of Phayao, Phayao 56000, Thailand, (metawee.faith@gmail.com)

Kavikumar Jacob, Department of Mathematics and Statistics, Faculty of Applied Sciences and Technology Universiti Tun Hussein Onn Malaysia, 86400 Malaysia. (kavi@uthm.edu.my)

Kifayat Ullah, Department of Mathematics and Statistics, International Islamic University Islamabad, Pakistan

M.B.Zeina, University of Aleppo, Aleppo-Syria (bisherzeina@alepuniv.edu.sy).

Fatimah Mahmood Mohammed, Department of Mathematic, College of Education for Pure Sciences, Tikrit University, Tikrit, IRAQ (dr.fatimahmahmood@tu.edu.iq)

Sadi Bayramov, Department of Algebra and Geometry, Baku State University, Baku, Azerbaijan (baysadi@gmail.com)

Qin Xin, Faculty of Science and Technology ,University of the Faroe Islands, Vestarabryggja 15, FO 100 Torshavn, Faroe Islands (QinX@setur.fo)

Darjan Karabašević, Faculty of Applied Management, Economics and Finance in Belgrade - MEF, University Business Academy in Novi Sad, Serbia

Dragisa Stanujkic, Technical Faculty in Bor, University of Belgrade, Bor, Serbia, (dstanujkic@tfbor.bg.ac.rs)

Aim and Scope

International Journal of Neutrosophic Science (IJNS) is a peer-review journal publishing high quality experimental and theoretical research in all areas of Neutrosophic and its Applications. IJNS is published quarterly. IJNS is devoted to the publication of peer-reviewed original research papers lying in the domain of neutrosophic sets and systems. Papers submitted for possible publication may concern with foundations, neutrosophic logic and mathematical structures in the neutrosophic setting. Besides providing emphasis on topics like artificial intelligence, pattern recognition, image processing, robotics, decision making, data analysis, data mining, applications of neutrosophic mathematical theories contributing to economics, finance, management, industries, electronics, and communications are promoted. Variants of neutrosophic sets including refined neutrosophic set (RNS). Articles evolving algorithms making computational work handy are welcome.

Topics of Interest

IJNS promotes research and reflects the most recent advances of neutrosophic Sciences in diverse disciplines, with emphasis on the following aspects, but certainly not limited to:

- Neutrosophic sets
- Neutrosophic algebra
- Neutrosophic topolog
- Neutrosophic graphs
- Neutrosophic probabilities
- Neutrosophic tools for decision making
- Neutrosophic theory for machine learning
- Neutrosophic statistics
- Neutrosophic numerical measures
- Classical neutrosophic numbers
- A neutrosophic hypothesis
- The neutrosophic level of significance
- The neutrosophic confidence interval
- The neutrosophic central limit theorem
- Neutrosophic theory in bioinformatics and medical analytics
- Neutrosophic tools for big data analytics
- Neutrosophic tools for deep learning
- Neutrosophic tools for data visualization
- Quadripartitioned single-valued neutrosophic sets
- Refined single-valued neutrosophic sets
- Applications of neutrosophic logic in image processing
- Neutrosophic logic for feature learning, classification, regression, and clustering

- Neutrosophic knowledge retrieval of medical images
- Neutrosophic set theory for large-scale image and multimedia processing
- Neutrosophic set theory for brain-machine interfaces and medical signal analysis
- Applications of neutrosophic theory in large-scale healthcare data
- Neutrosophic set-based multimodal sensor data
- Neutrosophic set-based array processing and analysis
- Wireless sensor networks Neutrosophic set-based Crowd-sourcing
- Neutrosophic set-based heterogeneous data mining
- Neutrosophic in Virtual Reality
- Neutrosophic and Plithogenic theories in Humanities and Social Sciences
- Neutrosophic and Plithogenic theories in decision making
- Neutrosophic in Astronomy and Space Sciences

Published and typeset in American Scientific Publishing Group (ASPG) is a USA academic publisher, established as LLC company on 2019 at New Orleans, Louisiana, USA. ASPG publishes online scholarly journals that are free of submission charges.

American Scientific Publishing Group (ASPG) LLC,
New Orleans, USA
Mailing Address: 625 Wright Ave, Gretna, LA 70056, USA
Phone: +1(504) 336-3385
Fax: +1-888-417-7465
e-mail: manager@americaspg.com
www.americaspg.com

Estimating the Ratio of a Crisp Variable and a Neutrosophic Variable

Carlos N. Bouza-Herrera[1,*], Mir Subzar[2]

[1] Universidad de La Habana La Habana , Cuba
Email: bouza@matcom.uh.cu
[2] Jammu and Kashmir Institute of mathematical sciences (JKIMS), J&K, India
Email: subzarstat@gmail.com

Abstract
The estimation of the ratio of two means is studied within the neutrosophic theory framework. The variable of interest Y is measured in a sample of units and the auxiliary variable X is obtainable for all units using records or predictions. They are correlated and the sample is selected using simple random sampling. The indeterminacy of the auxiliary variable is considered and is modeled as a neutrosophic variable. The bias and variance of the proposed estimator are derived.

Keywords Ratio of Means, Neutrosophic Variable, Bias, Mean Squared Errors, Taylor Series.

1. Introduction

Atanassov (1999), considered how the soft set theory constitutes a general mathematical tool for modeling uncertainty and impreciseness. This approach overcomes the need of parameterizing, as in the theories of probability, fuzzy sets and rough sets . The neutrosophic theory is based on a new set conceptualization. The roots of it may be found in Smarandache (2002, 2003). Neutrosophic set is a generalization of the so-called intuitionistic fuzzy sets. This theory is being used in different areas of knowledge. See recent examples as Ajay, (2020), Crespo Berti (2020) in modeling real life problems; Hatip (2020) and Saqlain et al. (2020) who developed extensions of it.

Neutrosophic theory may be used for characterizing the indeterminacy of a variable. In real life problems the parameters of a population are unknown, and statistical inferential procedures usually replace them by an adequate estimation. Therefore, the unknowledge of the value of a parameter is overcome by determining approximate values of it. The statistician knows that the obtained values are imprecise. The inaccuracy is measured by using some formulas that provide a confidence level or an estimation error of the estimation. See recent discussions in ratio

DOI: 10.5281/zenodo.4030309
Received: May 03, 2020 Accepted: septembre 02, 2020

estimation in Bouza and Alomari (2019), where solutions are derived by using auxiliary information for ranking the pre-selected units, and Subzar et al. (2019) who derived robust alternative ratio estimators.

Neutrosophic theory provides a new framework for dealing with impreciseness. It is well known that using a Neutrosophic point of view statistical concepts and methods may expanded, see Smarandache (2013, 2014), Schweizer (2020), Cacuango et al. (2020). for example. This theory deals not with crisp values of the variables, but with set values.

Neutrosophic statistical data analysis issues have been discussed in different papers, and alternative tools have been derived on the domain of neutrosophic sets. In general, Neutrosophic Statistics may be considered as an alternative to classical statistics. Decision makers may be concerned with applications where to deal with Neutrosophy in the sample is needed. See for example the papers of Hanafy, et al. (2012) who studied neutrosophic correlation in this context, Patro and Smarandache (2016) and Alhabib et al. (2018), who dealt with distributions. This paper is highly motivated by the contributions of Aslam (2018, 2019b), Aslam, et al (2020b), where sampling based tools for control charts neutrosophic random variables were developed as well as the applications of Neutrosophic statistical tools reported in Aslam (2019a) and Aslam et al. (2020a).

A basic structure in neutrosophic theory is the set

$$A_N = \{\langle \xi, T_A(\xi), I_A(\xi), F_A(\xi)\rangle | \xi \in \chi\};\ (\xi) \neq \phi \tag{1.1}$$

$$T_A(\xi) = degree\ of\ membership \tag{1.2}$$

$$I_A(\xi) = degree\ of\ indeterminacy, F_A(\xi) = degree\ of\ non\ membership \tag{1.3}$$

$$\upsilon_A(\xi)\chi \longrightarrow [0,1], \upsilon = T, I, F; \upsilon \in\ ]\ \ ^{-}0, 1^{+}[\tag{1.4}$$

From a practical point of view the interval [0,1] is used due to the difficulties that arise from using $]\ \ ^{-}0, 1^{+}[$.

For developing estimation methods in the context of this study neutrosophic numerical measures play a key role. The following results are particularly important.

Take $\forall\gamma \in R, Z \neq 0, Z \neq -W$ and holds that $\gamma Z + \gamma W I_Z = \gamma(Z + W I_Z)$. Then

1. $\forall\gamma \in R, Z \neq 0, Z \neq -W$ holds that $\frac{\gamma I_Z}{Z+WI_Z} = \frac{\gamma}{Z+W} I_Z$
2. $\forall Z \neq 0, Z \neq -W$ holds that $\frac{I_Z}{Z+WI_Z} = (Z + WI_Z)^{-1} I_Z$
3. $\forall Q \neq 0$ holds that $\frac{Z+WI_Z}{Q} = \frac{Z}{Q} + \frac{WI_Z}{Q}$
4. $\forall Z \neq 0, Z \neq -W$ holds that $\frac{Q}{Z+WI_Z} = \frac{Q}{Z} + \frac{QW}{Z(Z+W)} I_Z$

DOI: 10.5281/zenodo.4030309

See Smarandache (2013, 2014) for details.

The statistical procedures, based on equations and formulas, may be generalized, by using the framework provided by neutrosophic theory. Substituting an estimate of a parameter θ by a set value, say $\hat{\theta}_N$, the estimate determines a neighborhood of $\hat{\theta}$, which includes the point estimate $\hat{\theta}$. The impreciseness associated to the point estimator $\hat{\theta}$ is included in the neutrosophic representation of the estimate.

An open-minded statistician will agree with this modeling. In real life, a sample is selected from the population but when the statistics are computed the situation has changed. That is, the true value of the variable is different from the value measured and used in the computations. Consider for example a study of the persons infected with Covid-19. The researcher selects a sample for developing PSR-tests and the degree of infection of the selected persons is measured, say Z. When the laboratory ends processing the collection of tests, the real value of Z in the patients may be very different. Hence, the decision making rule should consider that the value Z is imprecise. Determining a neighborhood of the computed estimate would be more realistic. Referring only to randomness, for modeling uncertainty, is myopic in many cases. Developing particular neutrosophic statistical methods should allow dealing with randomness and indeterminacy at the same time.

Let us provide a theoretical frame for sampling. Take a finite population of items

$$U = \{u_1, \dots, u_M\} \tag{1.5}$$

A random sample is selected from U using a sample design d. The probabilistic model is characterized by $\{U, d\}, The$ sample space S is an adequate algebra. The sampler fixes to use as sampling design a certain probability measure

$$d(s) = probability\ of\ observing\ a\ subset\ s \subseteq U, \sum_{s \in S} d(s) = 1 \tag{1.6}$$

Taking Y as the variable of interest and u_i as an item of U the evaluation is represented by $Y(u_i)=Y_i$. A random sample s of size m is selected by using the sample design d. If $u_i \in s$ then ,Y_i is a random variable. In classical statistics Y is evaluated in the units selected and an estimate (a statistic) is computed. The unknown parameter θ is estimated using an estimator

$$\hat{\theta} = \hat{\theta}\big(Y(s)\big), Y(s) = \{Y_1, \dots, Y_m | \forall i = 1, \dots, m; u_i \in s \ \} \tag{1.7}$$

The behavior of a statistic is evaluated considering how it behaves in the long run. That is, how is its performance when analyzing a long sequence of samples $s_1, .., s_P, P \to \infty$.

DOI: 10.5281/zenodo.4030309

Considering neutrosophic statistics an extension of the classical statistics may be developed using the similar principles. When the statistician considers that the data is known only approximately , he/she would not be confident in the inferences generated using crisp numbers. For example if a question is sensitive the respondent will tend to falsify the true value of Y, if carrying the stigma. Then the data is expected to be indeterminate. This situation is present in different neutrosophic studies as in the problems analyzed in Cacuango et al. (2020).

Neutrosophic statistics framework admits that the information provided by the data is not crisp, but ambiguous, vague, imprecise and/or incomplete. When the indeterminacy is zero the analysis made using a neutrosophic point of view would coincide with the results derived by classical statistics. The practitioner, using neutrosophic statistical methods, would interpret and organize the data taking into account the existence of these indeterminacies for obtaining some clues on the underlying patterns.

Some basic ideas on the estimation within a neutrosophic theory framework are developed in the next section. Section 3 is concerned with the development of some aspects of estimation theory in a Neutrosophic context. Section 4 develops a theory on ratio estimation. Numerical experiments are discussed in the Section 5.

2. Estimation in a Neutrosophic context

The proposals of Smarandache (2014 , 2016) discussed how common statistic equations and formulas, due to the data indeterminacy of some of the involved variables, may be better replaced by considering that they take values in a fixed set. The usual notation is to replace the variable crisp Z by its neutrosophic counterpart Z_N. N identifies that the variable is "neutrosophic". The impreciseness on the true value of Z is modeled by considering not a value but a set including it. In the applications of statistics the decision makers frequently deal with imprecise data. A convenient model seems to be considering $Z_{iN} = Z_i + A_i I_Z$ instead of Z_i The statistician measures Z_i but has the feeling that it is imprecise. It is subject to a basic error, which belongs to an interval I_Z, and is "tuned" by A_i for each "i". For example, a promoter obtains in the web a value of the index of achievement of a singer, say Z_i , but doubts that it is correct, as changes in the public preferences are constant. Hence, the change in the singers indexes moves in I_Z =(-2,5 2,5) but for a particular singer the decision maker considers that it may be 3,5 times larger. Then, is recorded Z_i+3,5I_Z. Note that now the decision maker is able to implement decision rules where impreciseness is modeled.

Consider a sample of size m, the observations determine the sequence $\{(Z_i + A_i I_Z), i = 1, \dots m\}$. From Smarandache (2014 , 2016), is easily deduced that the sample mean is

$$\bar{Z}_{N(m)} = \bar{Z}_m + \bar{A}_m I_z = \frac{1}{m}\left(\sum_{i=1}^{m} Z_i + \sum_{i=1}^{m} A_i I_Z\right) \qquad (2.1)$$

The deviation of each observation is

DOI: 10.5281/zenodo.4030309

$$D_{N(m)i} = (Z_i + A_i I_Z) - (\bar{Z}_m + \bar{A}_m I_z) \quad (2.2)$$

and its square is given by

$$D^2_{N(m)i} = \left[D_{N(m)i} = (Z_i + A_i I_Z) - (\bar{Z}_m + \bar{A}_m I_z)\right]^2 \quad (2.3)$$

Therefore, the sample variance is

$$s^2_{Z_{N(m)}} = \frac{1}{m}\sum_{i=1}^{m}(Z_i - \bar{Z}_m)^2 + \frac{1}{m}\sum_{i=1}^{m}[2(Z_i - \bar{Z}_m)(A_i - \bar{A}_m) - (A_i - \bar{A}_m)^2]I_Z \quad (2.4)$$

because $(Z_i - \bar{Z}_m + A_i I_Z - \bar{A}_m I_z)^2 = (Z_i - \bar{Z}_m)^2 + [2(Z_i - \bar{Z}_m)(A_i - \bar{A}_m) + (A_i - \bar{A}_m)^2]I_Z$

(2.1) and (2.4) estimate the neutrosophic parameters

$$\bar{Z}_N = \bar{Z}_M + \bar{A}_M I_z = \frac{1}{M}\left(\sum_{i=1}^{M} Z_i + \sum_{i=1}^{M} A_i I_Z\right) \quad (2.5)$$

$$\sigma^2_{Z_{NM}} = \frac{1}{M}\sum_{i=1}^{M}(Z_i - \bar{Z}_M)^2 + \frac{1}{M}\sum_{i=1}^{M}[2(Z_i - \bar{Z}_M)(A_i - \bar{A}_M) - (A_i - \bar{A}_M)^2]I_Z \quad (2.6)$$

The interest of the decision maker is concerned with estimating a ratio of two variables. One of them is measured and the other is obtained from records. For example, we may develop an inquiry and obtain an achievement index for singers of the disk company. The records obtained in the web provide information on the downloads of a song but they are considered imprecise. Then the decision maker has a crisp variable, obtained in the inquiry, and a neutrosophic one obtained from the web.

In the sequel the sample design considered is Simple Random Sample With Replacement (SRSWR). The paper is concerned with the estimation of the ratio of a crisp variable and a neutrosophic one. The approximated bias and variance of the proposal are derived in the next section.

3. Some considerations on sampling

The importance of sampling experiments in different fields of applied sciences is one of the most important achievements of statistical inferences. The model considers the existence of a finite population U= $\{u_1, \dots, u_M\}$. The units are well identifiable. The researcher is interested in estimating a function of a variable Y. It is well defined for each individual of the finite population U. It is useful assuming that the experimenter have the knowledge of an auxiliary variable X for any individual of U $\{X(u_i) = X_i, i = 1, \dots, M\}$ but Y is unknown. The sampler may use the known values of X in the inference process. Under some mild conditions we may develop models, which yield more accurate results including the information provided by X. It is common that we should deal with indeterminacies in the values of X and is needed to determine how the model is affected. Hence, as the recorded values of X are imprecise an alternative is to use the neutrosophic number $X+AI_X$.

DOI: 10.5281/zenodo.4030309

The sample experiments are described as usual. For the population U there is a sample space S. The sampler selects a sample design d(s). It assigns a probability to each element of the sample space and allows determining the probability of selecting a certain unit of U by computing

$$P(u_i) = \sum_{\{s \in S | i \in s\}} d(s) \quad (3.1)$$

The variable of interest Y is measured in each selected unit, it is a random variable y_i that provides a result $Y(u_i)$. The sampler looks in the collection of recorded values of X, $X_1, \dots, X_M$ and obtains the corresponding random value x_i. Due to the existence of some indeterminacy is considered that it is the neutrosophic random variable

$$X_i + I_{X_i}, I_{X_i} = A_i I \ . \quad (3.2)$$

Then, in the study should be considered the existence of indeterminacy in the records and acknowledging this fact to work within a neutrosophic framework. Hence, taking an individual u_i of the population $u_i \rightsquigarrow (Y_i, X_i + I_{X_i}), I_X = A_i I$ is obtainable from it. As the unit is selected with probability $P(u_i)$ the gathered information is random. Using the sample design d the expectation of the result of the random experiment may be determined. Performing simple operations with neutrosophic numbers for an observation it is

$$E_d(y_i, x_i + I_{x_i}) = \sum_{j=1}^{M} (Y_j, X_j + A_j I_X \ ) P(u_j) = \left(\sum_{j=1}^{M} Y_j \, P(u_j), \sum_{j=1}^{M} X_j \, P(u_j) + \left(\sum_{j=1}^{M} A_j \, P(u_j) \right) \, I_X \right) = (\mu_{1Y}, \mu_{1X} + \mu_{1A} I_X \ ). \quad (3.2)$$

In the rest of the paper, without losing in generality, is considered that $A_j = A$ for any $j=1,\dots,M$.

The sampling design to be considered in this paper is Simple Random Sampling (SRS) Without replacement. It is defined, see Singh (2003), as

$$d(s) = \begin{cases} \frac{1}{\binom{M}{m}} \ if \ \|s\| = m \\ 0 \ otherwise \end{cases} \quad (3.3)$$

In that case

$$\forall i = 1, \dots, M, P(u_i) = \frac{m}{M} . \quad (3.4)$$

This result also holds if the selection is made with replacement. When M is sufficiently large the difference between selecting with or without replacement is negligible, in terms of the inferential processes, when

$$\frac{M-m}{M} \cong 1. \quad (3.5)$$

DOI: 10.5281/zenodo.4030309

A SRS sample is selected and are observed the realizations $\{y_i, x_i + A_i I_X \ , \ \forall i = 1, \dots, m\}$. Under SRS $P(u_i)$ is constant and using (3.2) the expectation is

$$E_d\big(y_i, x_i + I_{x_i}\big) = (\bar{Y},\ \bar{X} + \bar{A}I_X\ ), \bar{Z} = \frac{1}{M}\sum_{j=1}^{M} Z_j, Z = Y, X \tag{3.6}$$

because, for this design

$$\mu_{1Z} = \bar{Z} = \frac{1}{M}\sum_{j=1}^{M} Z_i;\ \ Z = X, Y, A \tag{3.7}$$

Using (2.6) is easily derived that the variance of $x_i + I_{x_i}$is given by

$$V_d\big(x_i + I_{x_i}\big) = \sigma^2_{X_M} = \frac{1}{M}\sum_{i=1}^{M}(X_i - \bar{X}_M)^2 + \frac{1}{M}\sum_{i=1}^{M}[2(X_i - \bar{X}_M)(A_i - \bar{A}_M) - (A_i - \bar{A}_M)^2]I_Z =$$

$$= \sigma^2_X + \tau_{X,A} I_X \tag{3.8}$$

Y is a crisp variable and its variance is

$$V_d(y_i) = \sigma^2_{Y_M} = \frac{1}{M}\sum_{i=1}^{M}(Y_i - \bar{Y}_M)^2 \tag{3.9}$$

4. A ratio estimator

Frequently statistical research must deal with the estimation of a ratio or use it for deriving an estimator of the mean or the total of a variable of interest Y. A concomitant , or auxiliary, variable *X,* correlated with Y, is known. The population ratio of them is $R = \frac{\bar{Y}}{\bar{X}}$. Consider that a SRS sample is drawn from the population. A naive estimator is $\hat{R} = \frac{\bar{y}}{\bar{x}}$, where $\bar{y}$ and $\bar{x}$ are the sample means of Y and X. The auxiliary variable is obtained, commonly, from records or predictions, which usually are outdated and/or subject to impreciseness. The impreciseness may be modeled adequately in the context of Neutrosophic Theory. Consider the neutrosophic number $+AI_X, I_X \in (a, b)$. The sampler may model the imprecise knowledge on X by determining that for every individual of the population a measurement error interval. For example, if the decision maker considers that the percent of tax under-report is between 0% and 20%, is fixing that $I_X \in (0, 0{,}2X)$.

The variable Y is measured by direct interviewing the individuals of the population. It is a non-neutrosophic number. In the previous example, R may be

- The rate of the mean of the preferences of the public for a song with respect to the monthly mean of downloads.
- The rate of the reported taxes with respect to the previous occasion payments.
- The ratio of the fuel consumption reports of a transport enterprise in two consecutive months.
- The ratio of monthly incomes of employees versus the use of their credit card.

DOI: 10.5281/zenodo.4030309

The classical sampling theory assumes the non-existence of impreciseness in X. The sampler, being uncertain on the preciseness of the values of X, fixes (x_0, x_1) and works with the neutrosophic value $X_N = X_i + A_i I_{X_i}; I_{X_i} \in (x_0, x_1), i = 1, \dots, M$. Take the crisp ratio of the means of two variables Y and X

$$R = \frac{\frac{1}{N}\sum_{i\in U} Y_i}{\frac{1}{N}\sum_{i\in U} X_i} = \frac{\bar{Y}}{\bar{X}} \quad (4.1)$$

X is known but is expected that it may change when the data is processed for computing. As the values of X come from imprecise recorded data, the decision maker fixes a conservative rule using $Q = \bar{Y}, Z = \bar{X}, W = \bar{X}$. In the context of neutrosophic R is denoted as

$$R = \frac{Q}{Z+WI_Z} = \frac{Q}{Z} + \frac{QW}{Z(Z+W)} I_Z = \frac{\bar{Y}}{\bar{X}}; Q = \bar{Y}, Z = \bar{X}, W = 0, I_Z \in [0,1]. \quad (4.2)$$

The alternative neutrosophic population ratio is

$$R_N = \frac{\bar{Y}}{\bar{X}+\bar{X}I_{\bar{X}}} = \frac{\bar{Y}}{\bar{X}} + \frac{\bar{Y}}{\bar{X}^2+\bar{X}} I_{\bar{X}} = R + R^* I_{\bar{X}} \quad (4.3)$$

The operator $\frac{\bar{Y}}{\bar{X}^2+\bar{X}} = R^*$ is used for modeling the indetermination present in the unknowledge of the true value of $\bar{X}$. Values of $I_{\bar{X}}$ close to zero model the decision maker`s confidence that $\bar{X}$ is the true expectation of X. In many occasions is adequate using

$$I_{\bar{X}} = \frac{1}{N}\sum_{i=1}^{N} I_{X_i} = I_X, I_X \in (x_0, x_1). \quad (4.4)$$

The decision maker usually fixes x_0=0.

Once a sample is selected the sample mean of X_N is computed

$$\bar{x}_N = \bar{x} + \bar{A}_m I_X;\ \bar{x} = \frac{1}{m}\sum_{i=1}^{n} x_i \quad (4.5)$$

as well as

$$\bar{y} = \frac{1}{m}\sum_{i=1}^{n} y_i \quad (4.6)$$

The need of estimating a ratio of two variables in this context suggests using $\bar{A}_m = \bar{x}$. Then, the neutrosophic estimator is

DOI: 10.5281/zenodo.4030309

$$\hat{R}_N = \frac{\bar{y}}{\bar{x}_N} = \frac{\bar{y}}{\bar{x}} + \frac{\bar{y}}{\bar{x}^2+\bar{x}} I_X = r + r^* I_X \tag{4.7}$$

Take the first term and analyze

$$r - R = \frac{\bar{y}}{\bar{x}} - \frac{\bar{Y}}{\bar{X}} \tag{4.8}$$

Consider that $\Delta_{\bar{z}} = \frac{\bar{z}-\bar{Z}}{\bar{Z}}, Z = X, Y$. As $|\Delta_{\bar{x}} < 1|$ with $lim_{t\to\infty} \Delta_{\bar{x}}^t = \infty$ is valid developing $\Delta_{\bar{x}}$in the Taylor Series

$$r \cong R(1+\Delta_{\bar{y}})(1+\Delta_{\bar{x}})^{-1} \cong R\left[1+\Delta_{\bar{y}} - \Delta_{\bar{x}} + \Delta_{\bar{x}}^2 - \Delta_{\bar{y}}\Delta_{\bar{x}} + O(\Delta_{\bar{x}})\right]$$

The expected values of the summands are $E_d(\Delta_{\bar{z}}) = 0, z = x, y$ $E_d(\Delta_{\bar{x}}^2) = \frac{\sigma_X^2}{n\bar{X}^2}, E_d(\Delta_{\bar{y}}\Delta_{\bar{x}}) = \frac{\sigma_{XY}}{n\bar{X}\bar{Y}}$ and is derived that

$$E_d(r-R) \cong \frac{\sigma_x^2}{n\bar{X}^2} - \frac{\sigma_{xy}}{n\bar{X}\bar{Y}} \tag{4.9}$$

For the second term

$$(r^* - R^*)I_X \cong \left(\Delta_{\bar{y}} + \Delta_{d\bar{x}}^2 - \Delta_{\bar{y}}\Delta_{d\bar{x}} + O(\Delta_{d\bar{x}})\right) I_X \tag{4.10}$$

where

$$\Delta_{D\bar{x}} = \frac{(\bar{x}^2+\bar{x}) - (\bar{X}^2+\bar{X})}{\bar{X}^2+\bar{X}} \tag{4.11}$$

The approximation is valid assuming that $\Delta_{D\bar{x}} \cong 0$ for a sufficiently large sample size. The first term of the expected value is $E_d(\Delta_{\bar{y}}) = 0$. Note that

$$\Delta_{D\bar{x}}^2 = \left(\frac{(\bar{x}^2-\bar{X}^2)+(\bar{x}-\bar{X})}{\bar{X}^2+\bar{X}}\right)^2 \tag{4.12}$$

Its design expectation is

$$E_d(\Delta_{D\bar{x}}^2) = \frac{1}{(\bar{X}^2+\bar{X})^2}\left[(\mu_{\bar{x}4} - \bar{X}^4) + (\mu_{\bar{x}3} - \bar{X}^3) + \frac{\sigma_X^2}{m}\left(1 - 2(\bar{X}^2+\bar{X})\right)\right] \tag{4.13}$$

On the other hand

$$\Delta_{\bar{y}}\,\Delta_{D\bar{x}} = \left(\frac{\bar{y}-\bar{Y}}{\bar{Y}}\right)\left(\frac{(\bar{x}^2+x)-(\bar{X}^2+\bar{X})}{\bar{X}^2+\bar{X}}\right) \tag{4.14}$$

and

$$E_d(\Delta_{\bar{y}}\,\Delta_{d\bar{x}}) \cong \frac{1}{m}\left(\frac{1}{\bar{Y}}\right)\left(\frac{1}{\bar{X}^2+\bar{X}}\right)\left(m\sigma_{\bar{y}\bar{x}^2} + \sigma_{YX} - \bar{Y}\sigma_X^2\right) \tag{4.15}$$

Hence we may state that

DOI: 10.5281/zenodo.4030309

$$Bias\ (\hat{R}_N) \cong \left[\frac{\sigma_X^2}{m\bar{X}^2} - \frac{\sigma_{XY}}{m\bar{X}\bar{Y}}\right] + \frac{1}{(\bar{X}^2+\bar{X})}\left[\left[(\mu_{\bar{x}4} - \bar{X}^4) + (\mu_{\bar{x}3} - \bar{X}^3) + \frac{\sigma_X^2}{m}\left(1 - 2(\bar{X}^2 + \bar{X})\right)\right] + \frac{1}{m}\left(\frac{1}{\bar{Y}}\right)\left(\frac{1}{\bar{X}^2+\bar{X}}\right)\left(m\sigma_{\bar{y}\bar{x}^2} + \sigma_{\bar{y}\bar{x}} - \bar{Y}\sigma_X^2\right)\right] I_X \quad (4.16)$$

Then, we have proved the following result.

Proposition 1. The expectation of the estimator $\hat{R}_N = \frac{\bar{y}}{\bar{x}_N} = \frac{\bar{y}}{\bar{x}} + \frac{\bar{y}}{\bar{x}^2+\bar{x}} I_X = r + r^* I_X$ of the neutrosophic ratio $R_N = \frac{\bar{Y}}{\bar{X}} + \frac{\bar{Y}}{\bar{X}^2+\bar{X}} I_{\bar{X}} = R + R^* I_{\bar{X}}$, when the sample is selected using simple random sampling, is approximately

$$E_d(\hat{R}_N) \cong R_N + \left[\frac{\sigma_X^2}{\bar{X}^2} - \frac{\sigma_{XY}}{m\bar{X}\bar{Y}}\right] + \frac{1}{(\bar{X}^2+\bar{X})}\left[\left[(\mu_{\bar{x}4} - \bar{X}^4) + (\mu_{\bar{x}3} - \bar{X}^3) + \frac{\sigma_X^2}{m}\left(1 - 2(\bar{X}^2 + \bar{X})\right)\right] + \frac{1}{m}\left(\frac{1}{\bar{Y}}\right)\left(\frac{1}{\bar{X}^2+\bar{X}}\right)\left(\sigma_{\bar{y}\bar{x}^2} + \sigma_{\bar{y}\bar{x}} - \bar{Y}\sigma_X^2\right)\right] I_X; \ \ \mu_{\bar{x}t} = E_d(\bar{X}^t), \quad (4.17)$$

when the sample size is sufficiently large for accepting that both $\Delta_{\bar{x}^2}$ and $\Delta_{\bar{x}}$ are approximately equal to zero for a large sample size m.

For deriving the Mean Squared Error (MSE) consider

$$\left(\hat{R}_N - R_N\right)^2 = \left(\frac{\bar{y}}{\bar{x}} + \frac{\bar{y}}{\bar{x}^2+\bar{x}} I_X - \frac{\bar{Y}}{\bar{X}} - \frac{\bar{Y}}{\bar{X}^2+\bar{X}} I_{\bar{X}}\right)^2 = (r - R + r^* I_X - R_N^* I_X)^2 = (r - R)^2 + (r^* I_X - R_N^* I_X)^2 + 2((r - R)(r^* I_X - R_N^* I_X)) \quad (4.18)$$

Developing the first term in taylor series is obtained

$$(r - R)^2 \cong \Delta_{\bar{y}}^2 + \Delta_{\bar{x}^2}^2 - 2\Delta_{\bar{y}}\, \Delta_{\bar{x}} \quad (4.19)$$

Hence

$$E_d(r - R)^2 \cong \frac{\sigma_Y^2}{m\bar{Y}^2} + \frac{\sigma_X^2}{m\bar{X}^2} - 2\frac{\sigma_{YX}}{m\bar{Y}\ \bar{X}} \quad (4.20)$$

The second term is

$$(r^* - R^*)^2 I_X \cong \left(\Delta_{\bar{y}}^2 + \Delta_{D\bar{x}}^2 - 2\Delta_{D\bar{y}}\Delta_{d\bar{x}} + O\left(\Delta_{D\bar{x}}\right)\right) I_X \quad (4.21)$$

and its expectation is given by

$$E_d((r^* - R^*)^2 I_X) \cong \left[\frac{\sigma_Y^2}{m\bar{Y}^2} + \frac{1}{(\bar{X}^2+\bar{X})^2}\left[(\mu_{\bar{x}4} - \bar{X}^4) + (\mu_{\bar{x}3} - \bar{X}^3) + \frac{\sigma_X^2}{m}\left(1 - 2(\bar{X}^2 + \bar{X})\right)\right] - 2\frac{1}{m}\left(\frac{1}{\bar{Y}}\right)\left(\frac{1}{\bar{X}^2+\bar{X}}\right)\left(\sigma_{\bar{y}\bar{x}^2} + \sigma_{YX} - \bar{Y}\sigma_X^2\right)\right] I_X \quad (4.22)$$

Developing the third term is obtained

$$((r-R)(r^* I_X - R_N^* I_X)) \cong (\Delta_{\bar{y}} - \Delta_{\bar{y}} + \Delta_{\bar{x}}^2 - \Delta_{\bar{y}}\Delta_{\bar{x}})(\Delta_{\bar{y}} + \Delta_{D\bar{x}}^2 - \Delta_{D\bar{y}}\Delta_{D\bar{x}}\ )I_X \cong (\Delta_{\bar{y}}^2 - \Delta_{\bar{y}}\Delta_{\bar{x}}\ )I_X \quad (4.23)$$

because only the terms of order t≤2 in the Taylor Series are considered as significant. Therefore

$$E_d((r-R)(r^* I_X - R_N^* I_X)) \cong \left(\frac{\sigma_Y^2}{m\bar{Y}^2} - \frac{\sigma_{YX}}{m\bar{Y}\ \ \bar{X}}\right) I_X \quad (4.24)$$

These results are used for proving the following proposition

Proposition 2. Under the set of hypothesis of proposition 1 the approximate MSE of

$$\hat{R}_N = \frac{\bar{y}}{\bar{x}_N} = \frac{\bar{y}}{\bar{x}} + \frac{\bar{y}}{\bar{x}^2 + \bar{x}} I_X = r + r^* I_X \quad (4.25)$$

is

$$\begin{aligned} MSE(\hat{R}_N) \cong & \left[\frac{\sigma_Y^2}{m\bar{Y}^2} + \frac{\sigma_X^2}{m\bar{X}^2} - 2\frac{\sigma_{YX}}{m\bar{Y}\ \ \bar{X}}\right] \\ & + \left[\left[\frac{2\sigma_Y^2}{m\bar{Y}^2} + \frac{1}{(\bar{X}^2+\bar{X})^2}\left[(\mu_{\bar{x}4} - \bar{X}^4) + (\mu_{\bar{x}3} - \bar{X}^3) + \frac{\sigma_X^2}{m}\left(1 - 2(\bar{X}^2 + \bar{X})\right)\right]\right.\right. \\ & \left.\left. - 2\frac{1}{m}\left(\frac{1}{\bar{Y}}\right)\left(\frac{1}{\bar{X}^2+\bar{X}}\right)\left(\sigma_{\bar{y}\bar{x}^2} + \sigma_{yx} - \bar{Y}\sigma_x^2\right) - \frac{\sigma_{YX}}{m\bar{Y}\ \ \bar{X}}\right] I_X\right] \\ & = M_C + M_N \qquad (4.26) \end{aligned}$$

Note that, if $I_X = 0$ the classical sampling results on ratio estimators are obtained.

5. Numerical studies

Only one of the multiple theoretical challenges of developing neutrosophic counterparts of sampling models is considered in this paper. The estimation of a ratio when the auxiliary variable is neutrosophic poses a set of theoretical problems to be solved for other classes of ratio estimators.

With the aims of illustrating the behavior of proposal, data obtained from four real life problems are analyzed numerically. They are:

P1. 230 persons with ages in the interval 15-35 were questioned on the preferences for 5 songs. The reports (Y) were measured in a scale 1-10. The mean of the daily downloads of the songs in the last 30 days was the auxiliary variable X. The manager of a record company is the decision maker.

DOI: 10.5281/zenodo.4030309

P2. The farmers tax-report was the variable of interest Y and X was the last month tax-payment. The population size was 450. The consensus of the specialists of the state office performed the role of the decision maker.

P3. The fuel consumption report of a fleet of 76 vehicles in two consecutive months was measured. Y was the consumption in the actual month and X the report in the previous month. The owner of the enterprise acted as decision maker.

P4. The monthly incomes of 117 employees of an enterprise was Y and X was the total amount of operations with their credit cards. The owner of the enterprise acted as decision maker.

The researchers had a complete knowledge of Y and X. Hence was possible to compute the values of the involved parameters. $\hat{R}_N$ was obtained from the sample results and compared with the true value of R_N . B random samples of size m were selected from each population and the accuracy of the estimates was measured computing

$$\alpha_{Pj} = \frac{1}{B}\sum_{s=1}^{B}|r - R|_s + \frac{1}{B}\sum_{s=1}^{B}|r^* - R^*|_s I_X, = \alpha_{Cj} + \alpha_{Nj}\ j = 1,..,4 \qquad (5.1)$$

Using the population information was computed (4.26) for each population

$$MSE(\hat{R}_N) \cong M_{Cj} + M_{Nj} = MSE_j; j = 1, \dots, 4 \qquad (5.2)$$

See the results of the study in Table 1.

Table 1. Results of the Monte Carlo experiments

Population	m	B	α_{Cj}	α_{Nj}	α_{Pj}	M_{Cj}	M_{Nj}	MSE_j
1	25	1000	0,324	0,072-0,250	0,396-0,774	1,561	6,331-7,117	7.892-8,678
2	50	1500	1,920	0,400-0.652	2,320-2,552	5,452	0,851-1,807	6,303-7,259
3	15	2600	2,690	0,841-1,741	3,531-4,431	8,785	6,263-11,549	15,048-20,334
4	25	2000	1,130	0,757-0,965	1,887-2,095	1,873	6,022- 7.625	7,895-8,498

A lecture of the lines of the previous table suggests that the samples averaged an absolute difference between the estimate and the true value, which take values in the corresponding fifth column. The MSE of the methods appears in the eighth column. The decision makers fixed the corresponding I_X. They considered that was obtained a good description of the impreciseness of the estimates rules by their appreciations.

Acknowledgements The authors thank Professors F. Smarandache and Said Broumi for their encouragement for developing some ideas on sampling based on neutrosophic theory and to the referees, whose suggestions allowed improving seriously the quality of this paper.

Funding: “This research received no external funding”

Conflicts of Interest: “The authors declare no conflict of interest.”

REFERENCES

[1] Ajay, D., J. Aldring, S. Abirami, D. Jeni Seles Martina, “A SVTrN-number approach of multi-objective optimisation on the basis of simple ratio analysis based on MCDM method”International Journal of Neutrosophic Science Vol. 5, pp. 16-28, 2020

[2] Alhabib , R., M. M. Ranna , H. Farah , A.A. Salama “Some Neutrosophic Probability Distributions”, Neutrosophic Sets and Systems, Vol. 22, pp 30-38, 2018.

[3] Atanassov K. T. “Intuitionistic Fuzzy Sets, Physica–Verlag” , Heidelberg, New York, 1999.

[4] Aslam, M. “A new sampling plan using neutrosophic process loss consideration”. Symmetry, Vol.10, 132, 2018.

[5] Aslam, M. “Neutrosophic analysis of variance application to university students”. Complex & Intelligent Systems Vol. 5, pp 403–407 (2019a)

[6] Aslam, M. “Control Chart for Variance Using Repetitive Sampling Under Neutrosophic Statistical Interval System”. IEEE Access, (2019b)

[7] Aslam, M. “Design of the Bartlett and Hartley tests for homogeneity of variances under indeterminacy environment”. Journal of Taibah University for Science, Vol. 14 , pp 6–10, 2020.

[8] Aslam A., O. H. Arif, 1 and R. A. K. Sherwani “New Diagnosis Test under the Neutrosophic Statistics An Application to Diabetic Patients”. BioMed Research International , 2020a, https//doi.org/10.1155/2020/2086185.

[9] Aslam, A., O. H. Arif, and R. A. K. Sherwani “A new attribute sampling plan using neutrosophic statistical interval method”. BioMed Research International” Vol 2020, pp 1-6, 2020b,

[10] Bouza, C. N. and A. Alomari Ranked Set Sampling. 65 Years Improving The Accuracy In Data Gathering. Elsevier, ISBN 978-0-12-815044-3, 2019.

[11] Cacuango Almeida, G.; J. Cadena Morillo ; M. Carrillo Palacio “Discernimiento e inferencia de la reinserción social en Ecuador, basada en conjuntos de números de 2-tuplas”. Inv. Operacional, Vol. 41, pp 637-646, 2020.

[12] Crespo Berti L. A. “Application of the neutrosophic system to tax havens with a criminal approach”, International Journal of Neutrosophic Science , Vol. 5, pp. 91-106, 2020.

[13] Hanafy, I.M. , A.A.Salama and K. Mahfouz “Correlation of Neutrosophic Data. International Refereed Journal of Engineering and Science”, Vol. 1, pp 39-43, 2012.

[14] Hatip A. “The Special Neutrosophic Functions” International Journal of Neutrosophic Science , Vol. 4, pp. 104-116, 2020.

[15] Patro S.K. and F. Smarandache “The Neutrosophic Statistical Distribution, More Problems, More Solutions “. Neutrosophic Sets and Systems, Vol. 12, pp 73-79, 2016.

[16] Saqlain M. , A. Hamza and S. Farooq “Linear and Non-Linear Octagonal Neutrosophic Numbers: Its Representation, $\boldsymbol{\alpha} - \boldsymbol{Cut}$ and Application”, International Journal of Neutrosophic Science ,Vol. 3, pp. 29-43, 2020

[17] Singh, S. “Advanced Sampling Theory with Applications. How Michael selected Amy”. Kluwer, Dordrecht 2003.

[18] Smarandache, F. Neutrosophy “A New Branch of Philosophy”, in Multiple valued logic, An International Journal, Vol. 8, pp 297-384, 2002.

[19] Smarandache F. “Neutrosophic set a generalization of the intuitionistic fuzzy sets” , In Proceedings of the third conference of the European Society for fuzzy logic and Technolgye, EUSFLAT, Septamper Zittau Geamany; Univ. of Applied Sciences at Zittau Goerlit Vol. 2, pp 141-146, 2003.

[20] Smarandache, F. “Introduction to Neutrosophic Measure, Neutrosophic Integral, and Neutrosophic Probability”, Sitech-Education Publisher, Craiova – Columbus, 2013.

[21] Smarandache, F. Introduction to Neutrosophic Statistics. Infinite Study. 2014.

[22] Smarandache, F. “Neutrosophic Overset, Neutrosophic Underset, and Neutrosophic Offset Similarly for Neutrosophic Over-/Under-/Off- Logic, Probability, and Statistics”, Pons Editions Brussels, Belgium, 2016.

[23] Subzar, M., C., N. Bouza, S. Maqbool, T A Raja and B. A. Para Robust ratio type estimators in simple random sampling using Huber m estimation Inv. Operacional Vol. 40, pp 201-209, 2019.

[24] Schweizer P. “The Natural Bases of Neutrosophy”, International Journal of Neutrosophic Science , Vol. 9, pp. 100-109, 2020

DOI: 10.5281/zenodo.4030309

A Novel Approach for Assessing the Reliability of Data Contained in a Single Valued Neutrosophic Number and its Application in Multiple Criteria Decision Making

Dragisa Stanujkic [1], Darjan Karabasevic [2,*], Florentin Smarandache [3] and Gabrijela Popovic [2]

[1] Technical Faculty in Bor, University of Belgrade, Serbia; dstanujkic@tfbor.bg.ac.rs
[2] Faculty of Applied Management, Economics and Finance, University Business Academy in Novi Sad, Belgrade, Serbia; darjan.karabasevic@mef.edu.rs, gabrijela.popovic@mef.edu.rs
[3] Dept. Math and Sciences, University of New Mexico, Gallup, NM, USA; smarand@unm.edu

* Correspondence: darjan.karabasevic@mef.edu.rs

Abstract

Multiple criteria decision making is one of the many areas where neutrosophic sets have been successfully applied to solve various problems so far. Compared to a fuzzy set, and similar sets, neutrosophic sets use more membership functions which makes them suitable for using complex evaluation criteria in multiple criteria decision making. On the other hand, the application of three membership functions makes evaluation somewhat more complex compared to evaluation using fuzzy sets. The reliability of the data used to solve a problem can have an impact on the selection of the appropriate solution/alternative. Therefore, this paper discusses an approach that can be used to assess the reliability of information collected by surveying respondents. The usability of the proposed approach is demonstrated in the numerical illustration of the supplier selection.

Keywords: neutrosophy, reliability, single-valued neutrosophic numbers, decision-making.

1. Introduction

Multiple criteria decision making (MCDM) started to emerge about 50 years ago, and until now it is used for solving a number of different decision-making problems in different fields. MCDM can be defined as making choices in the presence of multiple conflicting criteria [1-3]. Solving complex decision-making problems is usually associated with the need to use a larger number of criteria or use of more complex criteria that are later decomposed into sub-criteria [4-5]. However, an increase in the number of criteria, as well as sub-criteria, can be less desirable in cases where data should be collected by the survey [6].

Significant progress in using the MCDM methods for solving complex decision-making problems was made after Zadeh [7] proposed fuzzy sets, on which basis Bellman and Zadeh [8] proposed fuzzy MCDM [9-10]. Since then, many extensions of fuzzy sets theory have been developed, such as: interval-valued fuzzy sets [11], intuitionistic fuzzy sets [12] and bipolar fuzzy sets [13] . In 1999, Smarandache [14] introduced the concept of neutrosophic sets, as a generalization of the fuzzy sets theory and their extensions.

DOI: 10.5281/zenodo.4030337

Received: May 06, 2020 Accepted: August 24, 2020

So far, neutrosophic sets are successfully used in the area of multi-criteria decision-making. Many extensions of the MCDM methods are proposed based on the use of neutrosophic numbers, such as: neutrosophic AHP [15]; neutrosophic TOPSIS [1]; neutrosophic MULTIMOORA [16]; neutrosophic WASPAS [17]; neutrosophic PROMETHEE [18]; neutrosophic VIKOR [19]; neutrosophic ARAS [20]; neutrosophic GRA [21]; neutrosophic EDAS [22], and so forth. Besides, it is worth mentioning newly-developed approaches, such as: the importance of neutrosophic soft matrices in decision-making [23], interval-valued neutrosophic soft sets in decision-making [24], as well as ivnpiv-neutrosophic soft sets for decision-making [25]. In general, neutrosophy so far is used in solving a number of decision-making problems [26-32].

Fuzzy sets theory introduces partial membership to a set, expressed by membership function $\mu_{(x)}$, where membership function can have different forms, such as: bell-shaped, triangular, trapezoidal and singleton. Some other extensions of fuzzy sets theory introduced other membership functions such as: a non-membership function $\nu_{(x)}$, a positive membership function $\mu^{+}_{(x)}$ and a negative membership function $\nu^{+}_{(x)}$. Neutrosophic sets theory introduces three membership functions that can be used to describe belonging to a set, that is; truth membership, indeterminacy membership, falsity membership. That is why neutrosophic sets could be more suitable for evaluating complex phenomena, events and problems.

However, the use of three membership functions can make evaluation somewhat more complex compared to evaluation using fuzzy sets. Therefore Smarandache *et al.* [33] proposed an approach that can be used to assess the reliability of information collected by surveying respondents. This approach is reviewed again in this article, and a new approach for determining the reliability of information contained in single valued neutrosophic numbers is also presented.

Therefore, the remainder of the article is organized as follows: in Section 2 basic elements of neutrosophic sets and single-valued neutrosophic numbers are considered. In Section 3 approaches for ranking single valued neutrosophic numbers are considered, and in Section 4 a numerical illustration is given in order to demonstrate the proposed approach. Finally, conclusions are given.

2. Basic Elements of Neutrosophic Sets and Single Valued Neutrosophic Numbers

Definition 1. Let X be a nonempty set, with a generic element in X denoted by x. Then, the Neutrosophic Set (NS) A in X is as follows [14]:

$$A = \left\{ \left\langle x, T_A(x), I_A(x), F_A(x) \right\rangle \middle| x \in X \right\}, \tag{1}$$

with: $T_A : X \to]^{-}0,1^{+}[$; $I_A : X \to]^{-}0,1^{+}[$; $F_A : X \to]^{-}0,1^{+}[$ and ${}^{-}0 \le T_A(x) + I_A(x) + F_A(x) \le 3^{+}$

where: $T_A(x)$, $I_A(x)$ and $F_A(x)$ are the truth-membership function, the indeterminacy-membership function and the falsity-membership function, respectively.

Definition 2. Let X be a nonempty set. The Single Valued Neutrosophic Set (SVNS) A in X is as follows [14, 34]:

$$A = \left\{ \left\langle x, T_A(x), I_A(x), F_A(x) \right\rangle \middle| x \in X \right\}, \tag{2}$$

with: $T_A : X \to [0,1]$; $I_A : X \to [0,1]$; $F_A : X \to [0,1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Definition 3. For an SVNS A in X, the triple $< t_A, i_A, f_A >$ is called the Single Valued Neutrosophic Number (SVNN) [14, 34].

Definition 4. Let $X_i =< t_i, i_i, f_i >$ be a collection of SVNNs and $x =< t_x, i_x, f_x >$ a SVNN; then the Hamming distance $h_{(x)}$ between x and the ideal point $x^{+} =< t^{+}, i^{+}, f^{+} >=< \max_i t_i, \min_i i_i, \min_i f_i >$ is as follows:

$$h_{(x)} = \frac{1}{3}\left(|t^+ - t_x| + |i^+ - i_x| + |f^+ - f_x| \right). \tag{3}$$

Definition 5. Let $A_j = < t_j, i_j, f_j >$ be a collection of SVNNs and $W = (w_1, w_2, ..., w_n)^T$ be an associated weighting vector. Then the Single Valued Neutrosophic Weighted Average (SVNWA) operator of A_j is as follows [35]:

$$SVNWA(A_1, A_2, ..., A_n) = \sum_{j=1}^{n} w_j A_j = \left(1 - \prod_{j=1}^{n} (1 - t_j)^{w_j}, \prod_{j=1}^{n} (i_j)^{w_j}, \prod_{j=1}^{n} (f_j)^{w_j} \right). \tag{4}$$

where: w_j is the element j of the weighting vector, $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$.

3. Determining the Reliability of the Information Contained in Single Valued Neutrosophic Numbers

Smarandache *et al*. [33] proposed an approach for accessing the reliability of the information $r_{(x)}$ contained in a SVNN, as follows:

$$r_{(x)} = \frac{t - f}{1 + i}, \tag{5}$$

where: t, i, f denote the truth, the intermediacy and the falsity of information contained in SVNN $x = < t, i, f >$, $r \in [-1, 1]$.

Example: Assume that x =<0.9, 0.1, 0.3> is a SVNN. Then, the reliability of x is $r_{(x)} = \frac{0.9 - 0.3}{1 + 0.1} = 0.55$

In this approach, it is proposed to calculate reliability as follows:

$$r_{(x)} = \begin{cases} \frac{|t - f|}{t + i + f} & t + i + f \neq 0 \\ 0 & t + i + f = 0 \end{cases}, \tag{6}$$

where: $r \in [0,1]$.

Example: Assume that x =<0.9, 0.1, 0.3> is a SVNN. Then, the reliability of x is $r_{(x)} = \frac{|0.9 - 0.3|}{0.9 + 0.1 + 0.3} = 0.46$

One comparison of the reliability calculated using Eq. (5) and Eq. (6) for some characteristic values of t, i and f is shown in Table 1.

Table 1. The reliability calculated using Eq. (5) and Eq. (6)

t	i	f	Eq. (5)	Eq. (6)
1	0	0	1	1
0	0	1	-1	1
1	0	1	0	0
1	1	0	0.5	0.5
0	1	1	-0.5	0.5
1	1	1	0	0
0	0	0	0	0

It can be seen from Table 1, Eq. (5) provides values from the interval [-1, 1], with a value of zero being the least desirable. Equation (6) provides values from the interval [0, 1] where a higher value of the reliability function is more desirable.

4. A Numerical illustration

In order to briefly demonstrate the usability of the SVNNs for solving MCDM problems, an example of supplier selection is presented in this section. Assume that one company has to consider engaging with a new supplier. Therefore, a team of three experts if formed with the aim to select the most appropriate supplier from three alternatives, denoted as $A_1 - A_3$, on the basis on the following criteria:

- C_1 – Delivery,
- C_2 – Quality,
- C_3 – Flexibility,
- C_4 – Service, and
- C_5 – Price.

The ratings obtained from three experts are shown in Tables 1 to 3.

Table 2. The ratings obtained from the first of three experts

	C_1	C_2	C_3	C_4	C_5
A_1	<0.9, 0.10, 0.30>	<0.7, 0.2, 0.3>	<0.6, 0.0, 0.0>	<0.7, 0.0, 0.0>	<0.5, 0.0, 0.1>
A_2	<0.8, 0.00, 0.00>	<0.8, 0.0, 0.1>	<0.8, 0.0, 0.0>	<0.8, 0.0, 0.0>	<0.8, 0.0, 0.0>
A_3	<0.7, 0.00, 0.00>	<0.5, 0.0, 0.0>	<0.6, 0.0, 0.0>	<0.6, 0.0, 0.0>	<0.7, 0.2, 0.0>
A_4	<0.8, 0.10, 0.10>	<0.6, 0.0, 0.0>	<0.7, 0.0, 0.3>	<0.5, 0.2, 0.2>	<0.5, 0.0, 0.0>

Table 3. The ratings obtained from the second of three experts

	C_1	C_2	C_3	C_4	C_5
A_1	<0.6, 0.00, 0.10>	<0.7, 0.0, 0.1>	<0.6, 0.0, 0.0>	<0.5, 0.0, 0.0>	<0.2, 0.0, 0.9>
A_2	<0.8, 0.00, 0.30>	<0.6, 0.0, 0.1>	<0.7, 0.0, 0.0>	<0.8, 0.0, 0.2>	<0.1, 0.0, 0.8>
A_3	<0.7, 0.00, 0.30>	<0.8, 0.0, 0.0>	<0.7, 0.0, 0.0>	<0.6, 0.0, 0.4>	<0.3, 0.0, 0.2>
A_4	<0.6, 0.00, 0.20>	<0.7, 0.1, 0.2>	<0.7, 0.0, 0.0>	<0.6, 0.0, 0.4>	<0.5, 0.0, 0.1>

Table 4. The ratings obtained from the third of three experts

	C_1	C_2	C_3	C_4	C_5
A_1	<0.8, 0.20, 0.20>	<0.6, 0.0, 0.4>	<0.5, 0.0, 0.0>	<0.6, 0.0, 0.1>	<0.8, 0.0, 0.4>
A_2	<0.6, 0.10, 0.10>	<0.6, 0.2, 0.4>	<0.8, 0.1, 0.0>	<0.5, 0.0, 0.1>	<0.7, 0.0, 0.0>
A_3	<0.6, 0.00, 0.00>	<0.7, 0.0, 0.3>	<0.6, 0.1, 0.0>	<0.6, 0.0, 0.0>	<0.5, 0.1, 0.3>
A_4	<0.7, 0.00, 0.00>	<0.8, 0.0, 0.2>	<0.7, 0.0, 0.0>	<0.6, 0.0, 0.1>	<0.6, 0.0, 0.0>

The reliability of ratings obtained using Eq. (5) and Eq. (6) are shown in Tables 5 and 6. The average reliability of all ratings are also shown in Tables 5 and 6.

Table 5. The reliability of ratings obtained from the first expert using Eq. (5)

	C_1	C_2	C_3	C_4	C_5
A_1	0.55	0.33	0.60	0.70	0.40
A_2	0.80	0.70	0.80	0.80	0.80
A_3	0.70	0.50	0.60	0.60	0.58
A_4	0.64	0.60	0.40	0.25	0.50
				Avg	0.59

Table 6. The reliability of ratings obtained from the first expert using Eq. (6)

	C_1	C_2	C_3	C_4	C_5
A_1	0.46	0.33	1.00	1.00	0.67
A_2	1.00	0.78	1.00	1.00	1.00
A_3	1.00	1.00	1.00	1.00	0.78
A_4	0.70	1.00	0.40	0.33	1.00
				Avg	0.82

DOI: 10.5281/zenodo.4030337

The average reliability of responses obtained from all three decision makers, calculated using Eq. (6), are accounted for in Table 7.

Table 7. The average reliability of ratings obtained from all experts using Eq. (6)

	Reliability
E_1	0.82
E_2	0.65
E_3	0.69

As can be seen from Table 7, all three experts provide relatively consistent responses, and therefore their ratings can be used for further evaluation of alternatives. In contrast, if the average reliability of ratings obtained from a respondent has low value, his or her responses must be rejected from further evaluation of the alternatives or his or her responses must be re-considered again until adequate reliability is achieved.

A possible scenario of the evaluation of alternatives is discussed below. A group decision matrix, shown in Table 8, is constructed using Eq. (4) and the following weights $w_j = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$. The overall ratings are calculated using Eq. (4) and the following weighting wector $w_j = (0.19, 0.22, 0.19, 0.18, 0.22)$, as it is shown in Table 9. The ideal point is also shown in Table 9.

Table 8. The group decision matrix

	C_1	C_2	C_3	C_4	C_5
w_j	0.19	0.22	0.19	0.18	0.22
A_1	<0.80, 0.1, 0.20>	<0.67, 0.1, 0.28>	<0.57, 0.0, 0.00>	<0.61, 0.0, 0.03>	<0.57, 0.0, 0.62>
A_2	<0.75, 0.0, 0.14>	<0.68, 0.1, 0.21>	<0.77, 0.0, 0.00>	<0.73, 0.0, 0.10>	<0.62, 0.0, 0.41>
A_3	<0.67, 0.0, 0.11>	<0.69, 0.0, 0.11>	<0.64, 0.0, 0.00>	<0.60, 0.0, 0.16>	<0.53, 0.1, 0.18>
A_4	<0.71, 0.0, 0.10>	<0.71, 0.0, 0.14>	<0.70, 0.0, 0.11>	<0.57, 0.1, 0.24>	<0.54, 0.0, 0.03>

Table 9. The overall ratings and ideal point

	Overall ratings
A_1	<0.65, 0.00, 0.00>
A_2	<0.71, 0.00, 0.00>
A_3	<0.63, 0.00, 0.00>
A_4	<0.65, 0.00, 0.10>
A^+	<0.71, 0.00, 0.00>

Finally, the ranking results, obtained using Eq. (3), are encountered for in Table 10.

Table 10. The ranking results

	$h_{(i)}$	Rank
A_1	0.0063	2
A_2	0.0000	1
A_3	0.0092	3
A_4	0.0179	4

As can be seen from Table 10, the most appropriate alternative is alternative denoted as A_2.

5. Conclusion

Neutrosophic sets theory introduces three membership functions that is why single-valued neutrosophic numbers could be suitable for evaluating alternatives in relation to the complex evaluation criteria in multiple criteria decision making. However, the use of three membership functions can make evaluation somewhat complex especially when the evaluation is based on data collected by the survey.

DOI: 10.5281/zenodo.4030337

The reliability of the data used to solve a problem can have an impact on the final selection of the appropriate alternative. In this manuscript, an improved procedure for estimating the reliability of the collected data is proposed.

Therefore, Smarandache et al. [33] has proposed an approach that can be used to assess the reliability of information collected by surveying respondents.

Compared to the previous approach, in the new approach reliability and information belong to the interval [0, 1], unlike the previously proposed approach where reliability belongs to the interval [-1, 1], which makes new application easier for using.

By using the proposed procedure, the reliability of data could be easily determined. In this paper, the usability and efficiency of the proposed approach is successfully demonstrated on an illustrative example of the supplier selection.

Funding: The research presented in this article was done with the financial support of the Ministry of Education, Science and Technological Development of the Republic of Serbia, as part of the financing of scientific research at the University of Belgrade, Technical Faculty in Bor.

Conflicts of Interest: “The authors declare no conflict of interest.”

References

1. Karabašević, D., Stanujkić, D., Zavadskas, E. K., Stanimirović, P., Popović, G., Predić, B., & Ulutaş, A., “A Novel Extension of the TOPSIS Method Adapted for the Use of Single-Valued Neutrosophic Sets and Hamming Distance for E-Commerce Development Strategies Selection”, Symmetry, Vol 12(8), pp. 1263, 2020.
2. Bakir, M., Akan, Ş., Kiraci, K., Karabasevic, D., Stanujkic, D., & Popovic, G., “Multiple-Criteria Approach of the Operational Performance Evaluation in the Airline Industry: Evidence from the Emerging Markets”, Romanian Journal of Economic Forecasting, Vol 23(2), pp. 149-172, 2020.
3. Ivanov, B., Jevtić, M., Stanujkić, D., Karabašević, D., & Smarandache, F., “Evaluation of websites of IT companies from the perspective of IT beginners”, Bizinfo, Vol 9(2), pp. 1-9, 2018.
4. Stanujkic, D., Karabasevic, D., Zavadskas, E. K., Smarandache, F., & Brauers, W. K., “A bipolar fuzzy extension of the MULTIMOORA method”, Informatica, Vol 30(1), pp. 135-152, 2019.
5. Stanujkic, D., Zavadskas, E. K., Liu, S., Karabasevic, D., & Popovic, G., “Improved OCRA method based on the use of interval grey numbers”, Journal of Grey System, Vol 29(4), pp. 49-60, 2017.
6. Pamucar, D., Lukovac, V., Bozanic, D., & Komazec, N., “Multi-criteria FUCOM-MAIRCA model for the evaluation of level crossings: case study in the Republic of Serbia”, Operational Research in Engineering Sciences: Theory and Applications, Vol 1(1), pp. 108-129, 2018.
7. Zadeh, L. A., “Fuzzy sets. Information and control”, Vol 8(3), pp. 338-353, 1965.
8. Bellman, R. E., & Zadeh, L. A., “Decision-making in a fuzzy environment”, Management science, Vol 17(4), pp. 141–164, 1970.
9. Stanujkic, D., Zavadskas, E. K., Karabasevic, D., Urosevic, S., & Maksimovic, M., “ An approach for evaluating website quality in hotel industry based on triangular intuitionistic fuzzy numbers”, Informatica, Vol 28(4), pp. 725-748, 2017.
10. Stanujkić, D., Smarandache, F., Zavadskas, E. K., & Karabašević, D., “Multiple Criteria Evaluation Model Based on the Single Valued Neutrosophic Set”, Neutrosophic Sets and Systems, Vol 14, pp. 3-6, 2016.
11. Turksen, I. B., “Interval valued fuzzy sets based on normal forms”, Fuzzy sets and systems, Vol 20(2), pp. 191-210, 1986.
12. Atanassov, K., “Intuitionistic fuzzy sets”, Fuzzy Sets and Systems, Vol 20, pp. 87-96, 1986.
13. Lee, K.M., “Bipolar-valued fuzzy sets and their operations”, Proceedings of International conference on intelligent technologies – InTech'2000, December 12-14, 2000, Bangkok, Thailand, 307-312, 2000.
14. Smarandache, F., “A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic”, American Research Press: Rehoboth, DE, USA. pp. 141, 1999.

15. Bolturk, E., & Kahraman, C., "A novel interval-valued neutrosophic AHP with cosine similarity measure" Soft Computing, Vol 22(15), pp. 4941-4958, 2018.
16. Stanujkic, D., Zavadskas, E. K., Smarandache, F., Brauers, W. K., & Karabasevic, D., "A neutrosophic extension of the MULTIMOORA method", Informatica, Vol 28(1), pp. 181-192, 2017.
17. Bausys, R., Kazakeviciute-Januskeviciene, G., Cavallaro, F., & Usovaite, A., "Algorithm selection for edge detection in satellite images by neutrosophic WASPAS method", Sustainability, Vol 12(2), pp. 548, 2020.
18. Zavadskas, E. K., Bausys, R., Kaklauskas, A., & Raslanas, S., "Hedonic shopping rent valuation by one-to-one neuromarketing and neutrosophic PROMETHEE method", Applied Soft Computing, Vol 85, pp. 105832, 2019.
19. Abdel-Basset, M., Zhou, Y., Mohamed, M., & Chang, V., "A group decision making framework based on neutrosophic VIKOR approach for e-government website evaluation", Journal of Intelligent & Fuzzy Systems, Vol 34(6), pp. 4213-4224, 2018.
20. Liu, P., & Cheng, S., " An Extension of ARAS Methodology for Multi-criteria Group Decision-Making Problems within Probability Multi-valued Neutrosophic Sets", International Journal of Fuzzy Systems, Vol 21(8), pp. 2472-2489, 2019.
21. Biswas, P., Pramanik, S., & Giri, B. C., "GRA method of multiple attribute decision making with single valued neutrosophic hesitant fuzzy set information", New trends in neutrosophic theory and applications, pp. 55-63, 2016.
22. Karaşan, A., & Kahraman, C., "A novel interval-valued neutrosophic EDAS method: prioritization of the United Nations national sustainable development goals", Soft Computing, Vol 22(15), pp. 4891-4906, 2018.
23. Deli, I., & Broumi, S., "Neutrosophic soft matrices and NSM-decision making", Journal of Intelligent & Fuzzy Systems, Vol 28(5), pp. 2233-2241, 2015.
24. Deli, I., "Interval-valued neutrosophic soft sets and its decision making", International Journal of Machine Learning and Cybernetics, Vol 8(2), pp. 665-676, 2017.
25. Deli, I., Eraslan, S., & Çağman, N., "ivnpiv-Neutrosophic soft sets and their decision making based on similarity measure", Neural Computing and applications, Vol 29(1), pp. 187-203, 2018.
26. Ashraf, S., & Abdullah, S., "Decision Support Modeling For Agriculture Land Selection Based On Sine Trigonometric Single Valued Neutrosophic Information", International Journal of Neutrosophic Science (IJNS), Vol 9(2), pp. 60-73, 2020.
27. Martin, N., & Smarandache, F., "Plithogenic Cognitive Maps in Decision Making", International Journal of Neutrosophic Science (IJNS), Vol 9(1), pp. 9-21, 2020.
28. Edalatpanah, S. A., "A direct model for triangular neutrosophic linear programming", International journal of neutrosophic science (IJNS), Vol 1(1), pp. 19-28, 2020.
29. Gayathri, N., Helen, M., & Mounika, P., "Utilization of Jaccard Index Measures on Multiple Attribute Group Decision Making under Neutrosophic Environment", International Journal of Neutrosophic Science (IJNS), Vol 3(2), pp. 67-77, 2020.
30. Stanujkić, D., Smarandache, F., Zavadskas, E. K., & Karabašević, D., "An Approach to Measuring the Website Quality Based on Neutrosophic Sets", New Trends in Neutrosophic Theory and Applications, Vol II, pp. 40-50, 2017.
31. Urošević, S., Stanujkić, D., Karabašević, D., & Brzaković, P., "Using single valued neutrosophic set to select tourism development strategies in eastern Serbia", Economics of Agriculture, Vol 65(2), pp. 555-568, 2018.
32. Ivanov, B., Jevtić, M., Stanujkić, D., & Karabašević, D., "Evaluation of websites of IT companies from the perspective of it beginners", Bizinfo, Vol 9(2), pp. 1-9, 2018.
33. Smarandache, F., Stanujkic, D. & Karabasevic, D., "An approach for assessing the reliability of data contained in a single valued neutrosophic number", Proceedings of 4th International scientific conference Innovation as an initiator of the development – MEFkon 2018, 6th December, 2018, Belgrade, Serbia, pp. 80-86, 2018.
34. Wang, H., Smarandache, F., Zhang, Y. Q. & Sunderraman, R., "Single valued neutrosophic sets", Multispace and Multistructure, Vol 4, pp. 410-413, 2010.

DOI: 10.5281/zenodo.4030337

35. Şahin, R., "Multi-criteria neutrosophic decision making method based on score and accuracy functions under neutrosophic environment", arXiv preprint arXiv:1412.5202, 2014.

DOI: 10.5281/zenodo.4030337

Plithogenic Cubic Sets

S.P. Priyadharshini [1,*] , F. Nirmala Irudayam [2] and F. Smarandache [3]

[1,2] Dept of Mathematics, Nirmala College for Women, India; priyadharshini125@gmail.com , nirmalairudayam@ymail.com

[3] Dept. Math and Sciences, University of New Mexico, Gallup, NM, USA; smarand@unm.edu

* Correspondence: priyadharshini125@gmail.com

Abstract

In this article, using the concepts of cubic set and plithogenic set, the ideas of plithogenic fuzzy cubic set, plithogenic intuitionistic fuzzy cubic set, Plithogenic neutrosophic cubic set are introduced and its corresponding internal and external cubic sets are discussed with examples. Primary properties of the Plithogenic neutrosophic cubic sets were also discussed.This concept is extremely suitable for addressing problems involving multiple attribute decision making as this plithogenic neutrosophic set are described by four or more value of attributes and the accuracy of the result is also so precise.

Keywords: Fuzzy set, Intuitionistic fuzzy set, Neutrosophic set, Cubic set, Plithogenic set, Plithogenic fuzzy cubic set, Plithogenic Intuitionistic fuzzy cubic set, Plithogenic neutrosophic cubic set.

1. Introduction

Zadeh implemented Fuzzy Sets[18].In [18] Zadeh gave the perception of a fuzzy set by an interval-valued fuzzy set , i.e. a fuzzy set with an intervalvalued belonging function. In conventional fuzzy concepts,numbers from the interval [0,1] are used to reflect, e.g., the degree of conviction of the expert in various statements.It is always tough for an expert to precisely enumerate their certainty; thus, rather than a particular value, it is more fitting to reflect this certainty degree by an interval or even by a fuzzy set. We obtain an interval valued fuzzy sets which were frequently used in real-life scenario.

Atanassov[2] introduced intuitionistic fuzzy sets(IFS), which is the generalisation of the fuzzy set. In IFS,each element is attached to both belonging and non-belonging grade with the constraint that the sum of these two grades is less than or equal to unity. If available knowledge is not sufficient to identify the inaccuracy of traditional fuzzy sets, IFS architecture can be viewed as an alternative / appropriate solution. Later on IFS were expanded to interval-valued IFS.

Smarandache [9, 10, 11] proposed Neutrosophic sets (NSs), a generalisation of FS and IFS, which is highly helpful for dealing with inadequate, uncertain, and varying data that exists in the real life. NSs are characterised by functions of truth (T), indeterminacy (I) and falsity (F) belonging functions. This concept is very essential in several areas of application since indeterminacy is clearly enumerated and the truth, indeterminacy, and falsity membership functions are independent.

DOI: 10.5281/zenodo.4030378

Received: May 11, 2020 Accepted: August 28, 2020

.Wang, Smarandache, Zhang and Sunderraman[17] proposed the definition of the interval valued neutrosophic set (INS) as an extension of the neutrosophic set.The INS could reflect indeterminate, inaccurate, inadequate and unreliable data that occurs in the reality.

Plithogeny is the origin, creation, production and evolution of new objects from the synthesis of conflicting or non-conflicting multiple old objects. Smarandache[12] introduced the plithogenic set as a generalisation of neutrosophy in 2017.

The elements of plithogenic sets are denoted by one or many number of attributes and each of it have several values.Each values of attribute has its respective (fuzzy, intuitionistic fuzzy or neutrosophic) appurtenance degree for the element x (say) to the plithogenic set P (say) with respect to certain constraints. For the first time, Smarandache[12] introduced the contradictory (inconsistency) degree between each value of attribute and the dominant value of attribute which results in getting the better accuracy for the plithogenic aggregation operators(fuzzy, intuitionistic fuzzy or neutrosophic).

Y.B. Jun et al [4] implemented a cubic set which is a combination of a fuzzy set with an interval valued fuzzy set. Internal and external cubic sets were also described and some properties were studied.

Y.B.Jun, Smarandache and Kim [4] introduced Neutrosophic cubic sets and the concept of internal and external for truth, falsity and indeterminacy values.Furthermore they provided so many properties of P (R) -union,P (R) –intersection for internal and external neutrosophic cubic sets.

In this article, using the principles of cubic sets and plithogenic set, we presented the generalisation of plithogenic cubic sets for fuzzy, Intuitionistic fuzzy and neutrosophic sets

2. Preliminaries

Definition 2.1 [15] Let Z be a universe of discourse and the fuzzy set $F=\{\langle z,\mu_f(z)\rangle \mid z\in Z\}$ is described by a belonging function μ_f as , $\mu_f : Z \to [0,1]$.

Definition 2.2 [10] Let Y be a non-empty set. Then, an interval valued fuzzy set B over Y is defined as $B=\{[B^-(y),B^+(y)]/y : y\in Y\}$ where $B^-(x)$ and $B^+(x)$ are stated as the inferior and superior degrees of belonging $y\in Y$ where $0\le B^-(y)+ B^+(y)\le 1$ correspondingly.

Definition 2.3 [2, 3] Let N be a non empty set.The set $A=\{\langle n,\lambda_A,\phi_A \rangle n\in N\}$ is called an intutionistic fuzzy set (in short, IFS) of N where the function $\lambda_A : N\to[0,1]$, $\phi_A : N\to[0,1]$ denotes the membership degree (say $\lambda_A(n)$) and non-membership degree (say $\phi_A(n)$) of each element $n\in N$ to the set A and satisfies the constraint that $0\le \lambda_A(n)+\phi_A(n)\le 1$.

Definition 2.4 [10] Let Y be a non-empty set. The set $A=\{< y,M_A(y),N_A(y) > \mid y\in Y\}$ is called an interval valued intuitionistic fuzzy sets (IVIFS) A in Y where the functions $M_A(y):Y\to[0,1]$ and $N_A(y):Y\to[0,1]$ denotes the degree of belonging, non-belonging of A respectively. Also $M_A(y)=[M_{AL}(y),M_{AU}(y)]$ and $N_A(y)=[N_{AL}(y),N_{AU}(y)]$, $0\le M_{AU}(y)+M_{AU}(y)\le 1$ for each $y\in Y$

Definition 2.5 [6] Let N be a non empty set.The set $A=\{\langle n,\lambda_A,\phi_A,\gamma_A\rangle n\in N\}$ is called a neutrosophic set (in short, NS) of N where the function $\lambda_A : N\to[0,1]$, $\phi_A : N\to[0,1]$ and $\gamma_A : N\to[0,1]$ denotes the membership degree (say $\lambda_A(n)$), indeterminacy degree (say $\phi_A(n)$), and non- membership degree (say $\gamma_A(n)$)of each element $n\in N$ to the set A and satisfies the constraint that $0\le \lambda_A(n)+\phi_A(n)+\gamma_A(n)\le 3$.

DOI: 10.5281/zenodo.4030378

Definition 2.6 [1] Let R be a non-empty set. An interval valued neutrosophic set (INS) A in R is described by the functions of the truth-value (A_T) , the indeterminacy (A_I) and the falsity-value (A_F) for each point $r \in R$, $A_T(r), A_I(r), A_F(r) \subseteq [0,1]$.

Definition 2.7 [4,5] Let E be a non-empty set. By a cubic set in E ,we construct a set which has the form $\Psi = \{< e, B(e), \mu(e) >| e \in E\}$ in which B is an interval valued fuzzy set (IVFS) in E and μ is a fuzzy set in E.

Definition 2.8 [4] Let E be a non-empty set. If $B^-(e) \le \mu(e) \le B^+(e)$ for all $e \in E$ then the cubic set $\Psi =< B, \mu >$ in E is called an internal cubic set (briefly IPS).

Definition 2.9 [4] Let E be a non-empty set. If $\mu(e) \notin (B^-(e), B^+(e))$ for all $e \in E$ then the cubic set $\Psi =< B, \mu >$ in E is called an external cubic set (briefly ECS)

Definition 2.10 [10] Plithogenic Fuzzy set for an interval valued (IPFS) is defined as $\forall c \in C, d : C \times W \to C([0,1])$, $\forall w \in W$ and $d(c, w)$ is an open, semi-open, closed interval included in [0, 1] and C([0, 1]) is the power set of the unit interval [0,1] (i.e) all subsets of [0,1].

Definition 2.11 [10] A set which has the form $\forall c \in C, d : C \times W \to C([0,1]^2)$ and $\forall w \in W$ is called interval valued plithogenic intuitionistic fuzzy set (IPIFS) where $d(c, w)$ is an open, semi-open, closed for the interval included in $[0,1]$.

Definition 2.12 [10] A set which has the form $\forall c \in C, d : C \times W \to C([0,1]^3)$ and $\forall w \in W$ is called interval valued plithogenic neutrosophic set (IPNS) where $d(c, w)$ is an open, semi-open, closed for the interval included in $[0,1]$.

3. Plithogenic Cubic sets

3.1 Plithogenic fuzzy Cubic sets

Definition 3.1.1 For a non empty set Y, the Plithogenic fuzzy cubic set (PFCS) is defined as $\Lambda = \{< y, B(y), \lambda(y) >| y \in Y\}$ in which B is an interval valued plithogenic fuzzy set in Y and λ is a fuzzy set in Y.

Example 3.1.2 The following values of attribute represents the criteria “Musical instruments” : Piano (the dominant one), guitar, saxophone, violin.

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Piano	Guitar	Saxophone	Violin
Appurtenance Degree $B(y)$	[0.5, 0.6]	[0.2, 0.8]	[0.4, 0.7]	[0.1, 0.3]
$\lambda(y)$	0.3	0.4	0.6	0.2

Definition 3.1.3 For a non-empty set Y, the PFCS $\Lambda =< B, \lambda >$ in Y is called an internal plithogenic fuzzy cubic set (briefly IPFCS) if ${B_{d_i}}^-(y) \le \lambda_i(y) \le {B_{d_i}}^+(y)$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes.

Example 3.1.4 The following values of attribute represents the criteria “Color” : Yellow(the dominant one), green, orange and red.

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Yellow	Green	Orange	Red

DOI: 10.5281/zenodo.4030378

Appurtenance Degree $B(y)$	[0.2, 0.3]	[0.6, 0.8]	[0.3, 0.6]	[0.4, 0.9]
$\lambda(y)$	0.2	0.7	0.5	0.8

Definition 3.1.5 For a non-empty set Y, the PFCS $\Lambda = < B, \lambda >$ in Y is called an external plithogenic fuzzy cubic set (briefly EPFCS) if $\lambda_i(y) \notin (B_{d_i}^{-}(y), B_{d_i}^{+}(y))$ for all $y \in Y$ and and d_i represents the contradictory degree and their respective value of attributes.

Example 3.1.6 The following values of attribute represents the criteria "Subjects" : Mathematics(the dominant one), Physics, Chemistry and Biology

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Mathematics	Physics	Chemistry	Biology
Appurtenance Degree $B(y)$	[0.4, 0.6]	[0.5, 0.8]	[0.2, 0.3]	[0.5, 0.9]
$\lambda(y)$	0.7	0.1	0.6	0.3

Remark 3.1.7 If any one of the attribute value $\lambda_i(y) = y$ for all $y \in Y$ and *i* represents the values of attribute then Λ is neither an IPFCS nor an EPFCS.

3.2 Plithogenic Intuitionistic fuzzy cubic set

Definition 3.2.1 For a non empty set Y, the Plithogenic Intuitionistic fuzzy cubic set (PIFCS) is defined as $\Lambda = \{< y, B(y), \lambda(y) > \mid y \in Y\}$ in which *B* is an interval valued Plithogenic Intuitionistic fuzzy set in Y and λ is a intuitionistic fuzzy set in Y.

Example 3.2.2 The following values of attribute represents the criteria "Mobile phone brands" : Apple (the dominant one), Samsung, Nokia, Lenova.

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Apple	Samsung	Nokia	Lenova
Appurtenance Degree $B(y)$	([0.2, 0.5], [0.3, 0.6])	([0.5, 0.8], [0.1, 0.7])	([0.4, 0.7], [0.2, 0.6])	([0.1, 0.3], [0.4,0.7])
$\lambda(y)$	([0.1,0.7])	([0.2, 0.4])	([0.1, 0.8])	([0.2, 0.5])

Definition 3.2.3 For a non-empty set Y, the PIFCS $\Lambda = < B, \lambda >$ in Y is called an internal plithogenic intuitionistic fuzzy cubic set (briefly IPIFCS) if $B_{d_i}^{-}(y) \le \lambda_i(y) \le B_{d_i}^{+}(y)$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes.

Example 3.2. The following values of attribute represents the criteria " Proficiency " : Excellent (the dominant one), good, average, poor

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Excellent	Good	Average	Poor
Appurtenance Degree $B(y)$	([0.3, 0.5], [0.1, 0.6])	([0.4, 0.6], [0.5, 0.8])	([0.1, 0.8], [0.7, 0. 8])	([0.2, 0.3], [0.1,0.6])
$\lambda(y)$	([0.4,0.5])	([0.6, 0.7])	([0.2, 0.5])	([0.3, 0.5])

Definition 3.2.5 For a non-empty set Y, the PIFCS $\Lambda = < B, \lambda >$ in Y is called an external plithogenic intuitionistic fuzzy cubic set (briefly EPIFCS) if $\lambda_i(y) \notin (B_{d_i}^{-}(y), B_{d_i}^{+}(y))$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes.

Example 3.2.6 The following values of attribute represents the criteria " Mode of Transport" : Bus (the dominant one), Car, Lorry, Train

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Bus	Car	Lorry	Train
Appurtenance Degree $B(y)$	([0.4, 0.6], [0.3, 0.5])	([0.5, 0.7], [0.2, 0.4])	([0.2, 0.6], [0.3, 0.7])	([0.3, 0.6], [0.1,0.4])
$\lambda(y)$	([0.1,0.6])	([0.4, 0.5])	([0.7, 0.2])	([0.7, 0.6])

3.3 Plithogenic Neutrosophic cubic set

Definition 3.3.1 Let Ω be an universal set and Y be a non empty set. The structure $\Lambda = \{< y, B(y), \lambda(y) > | \, y \in Y\}$ is said to be Plithogenic Neutrosophic cubic set (PNCS) in Y, where $B = \left\{\left(B_{d_i}{}^{T}(y), B_{d_i}{}^{I}(y), B_{d_i}{}^{F}(y)\right)\right\}$ is an interval valued Plithogenic Neutrosophic set in Y and $\lambda = \{(\lambda_i^{T}(y), \lambda_i^{I}(y), \lambda_i^{F}(y)\}$ is a neutrosophic set in Y.

The pair $\Lambda = < B, \lambda >$ is called plithogenic neutrosophic cubic set over Ω where Λ is a mapping given by $\Lambda : B \to NC(\Omega)$ .The set of all plithogenic neutrosophic cubic sets over Ω will be denoted by P_N^{Ω}

Example 3.3.2 consider the attribute "Size" which has the following values: Small (the dominant one), medium, big, very big.

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Small	Medium	Big	Very big
Appurtenance Degree $B(y)$	([0.3, 0.5], [0.1, 0.8], [0.1, 0.9])	([0.1, 0.3], [0.4, 0.8], [0.4, 0.9])	([0.1, 0.5], [0.2, 0. 6], [0.6, 0.9])	([0.2, 0.6], [0.2,0.5], [0.7, 0.9])
$\lambda(y)$	([0.4,0.5, 0.8])	([0.3, 0.7, 0.4])	([0.2,0.5,0.8])	([0.5,0.2,0.8])

Definition 3.3.3 For a non-empty set Y, the plithogenic neutrosophic cubic set $\Lambda = < B, \lambda >$ in Y is called

(i) truth internal if $B_{d_i}{}^{-T}(y) \le \lambda_i^{T}(y) \le B_{d_i}{}^{+T}(y)$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes. (3.1)

(ii) indeterminacy internal if $B_{d_i}{}^{-I}(y) \le \lambda_i^{I}(y) \le B_{d_i}{}^{+I}(y)$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes. (3.2)

(iii) falsity internal if $B_{d_i}{}^{-F}(y) \le \lambda_i^{F}(y) \le B_{d_i}{}^{+F}(y)$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes. (3.3).

If a plithogenic neutrosophic cubic set in Y satisfies the above equations we conclude that Λ is an internal plithogenic neutrosophic cubic set (IPNCS) in Y.

Example 3.3.4 The following values of attribute represents the criteria "Sports": Volley ball(the dominant one), Basket ball, Cricket, Bat-minton

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Volley ball	Basket ball	Cricket	Bat-minton

DOI: 10.5281/zenodo.4030378

Appurtenance Degree $B(y)$	([0.2, 0.5], [0.6, 0.8], [0.1, 0.5])	([0.1, 0.3], [0.4, 0.5], [0.6, 0.9])	([0.5, 0.8], [0.1, 0. 4], [0.6, 0.9])	([0.3, 0.6], [0.1,0.5], [0.7, 0.9])
$\lambda(y)$	([0.3,0.6, 0.4])	([0.2, 0.5, 0.8])	([0.7,0.2, 0.8])	([0.4, 0.3, 0.9])

Definition 3.3.5 For a non-empty set Y, the plithogenic neutrosophic cubic set $\Lambda = <B, \lambda>$ in Y is called

(i) truth external if $\lambda_i^T(y) \notin (B_{d_i}^{-T}(y), B_{d_i}^{+T}(y))$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes. (3.4)

(ii) indeterminacy external if $\lambda_i^I(y) \notin (B_{d_i}^{-I}(y), B_{d_i}^{+I}(y))$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes. (3.5)

(ii) falsity external if $\lambda_i^F(y) \notin (B_{d_i}^{-F}(y), B_{d_i}^{+F}(y))$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes. (3.6)

If a plithogenic neutrosophic cubic set in Y satisfies the above equations, we conclude that Λ is an external plithogenic neutrosophic cubic set (EPNCS) in Y.

Example 3.3.6 The following values of attribute represents the criteria "Seasons" : Spring (the dominant one), Winter, Summer, Autumn

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Spring	Winter	Summer	Autumn
Appurtenance Degree $B(y)$	([0.3, 0.6], [0.7, 0.5], [0.4, 0.8])	([0.1, 0.3], [0.2, 0.3], [0.6, 0.8])	([0.1, 0.4] , [0.2, 0. 6], [0.8, 0.9])	([0.3, 0.5], [0.2,0.7], [0.6, 0.8])
$\lambda(y)$	([0.1,0.2, 0.9])	([0.4, 0.5, 0.9])	([0.7, 0.9, 0.6])	([0.1, 0.8, 0.3])

Theorem 3.3.7 Let Y be a non empty set and $\Lambda = <B, \lambda>$ be a PNCS in Y which is not external. Then there exists $y \in Y$ such that $\lambda_i^T(y) \in (B_{d_i}^{-T}(y), B_{d_i}^{+T}(y))$, $\lambda_i^I(y) \in (B_{d_i}^{-I}(y), B_{d_i}^{+I}(y))$ or $\lambda_i^F(y) \in (B_{d_i}^{-F}(y), B_{d_i}^{+F}(y))$ where d_i represents the contradictory degree and their respective value of attributes..

Proof. Direct proof

Theorem 3.3.8 Let Y be a non empty set and $\Lambda = <B, \lambda>$ be a PNCS in Y. If Λ is both T-internal and T-external, then $(\forall y \in Y)\left(\lambda_i^T(y) \in \{B_{d_i}^{-T}(y) \mid y \in Y\} \cup \{B_{d_i}^{+T}(y) \mid y \in Y\}\right)$ where d_i represents the contradictory degree and their respective value of attributes.

Proof. The conditions (3.1) and (3.4) implies that $B_{d_i}^{-T}(y) \le \lambda_i(y) \le B_{d_i}^{+T}(y)$ and $\lambda_i^T(y) \notin (B_{d_i}^{-T}(y), B_{d_i}^{+T}(y))$ for all $y \in Y$ and i = 1, 2, 3,…n.

Then it indicates that $\lambda_i^T(y) = B_{d_i}^{-T}(y)$ or $\lambda_i^T(y) = B_{d_i}^{+T}(y)$, and hence

$\lambda_i^T(y) \in \{B_{d_i}^{-T}(y) \mid y \in Y\} \cup \{B_{d_i}^{+T}(y) \mid y \in Y\}$ where d_i represents the contradictory degree and their respective value of attributes.

Hence the proof.

DOI: 10.5281/zenodo.4030378

Correspondingly the subsequent theorems hold for the indeterminate and falsity values.

Theorem 3.3.9 Let Y be a non empty set and $\Lambda = <B, \lambda>$ be a PNCS in Y. If Λ is both I-internal and I-external, then $(\forall y \in Y)\left(\lambda_i^{I}(y) \in \left\{B_{d_i}^{-I}(y) \mid y \in Y\right\} \cup \left\{B_{d_i}^{+I}(y) \mid y \in Y\right\}\right)$ where d_i represents the contradictory degree and their respective value of attributes.

Theorem 3.3.10 Let Y be a non empty set and $\Lambda = <B, \lambda>$ be a PNCS in Y. If Λ is both *F*-internal and *F*-external, then $(\forall y \in Y)\left(\lambda_i^{F}(y) \in \left\{B_{d_i}^{-F}(y) \mid y \in Y\right\} \cup \left\{B_{d_i}^{+F}(y) \mid y \in Y\right\}\right)$ where d_i represents the contradictory degree and their respective value of attributes.

Definition 3.3.11 Let Y be a non empty set, The complement of $\Lambda = <B, \lambda>$ is said to be the PNCS $\Lambda^c = <B^c, \lambda^c>$ where $B^c = \left\{\left(B^c{}_{d_i}^{T}(y), B^c{}_{d_i}^{I}(y), B^c{}_{d_i}^{F}(y)\right)\right\}$ is an interval valued PNCS in Y and $\lambda = \{(\lambda^c{}_i^{T}(y), \lambda^c{}_i^{I}(y), \lambda^c{}_i^{F}(y)\}$ is a neutrosophic set in Y.

Theorem 3.3.12 Let Y be a non empty set and $\Lambda = <B, \lambda>$ be a PNCS in Y. If Λ is both *T*-internal and *T*-external, then the complement $\Lambda^c = <B^c, \lambda^c>$ of $\Lambda = <B, \lambda>$ is an *T*-Internal and *T*-external PNCS in Y.

Proof. Let Y be a non empty set .If $\Lambda = <A, \lambda>$ is an *T*-internal and *T*-external PNCS in Y, then $B_{d_i}^{-T}(y) \le \lambda_i(y) \le B_{d_i}^{+T}(y)$ and $\lambda_i^{T}(y) \notin (B_{d_i}^{-T}(y), B_{d_i}^{+T}(y))$ for all $y \in Y$ and i = 1, 2, 3,…,n. It follows that $1 - B_{d_i}^{-T}(y) \le 1 - \lambda_i(y) \le 1 - B_{d_i}^{+T}(y)$ and. $1 - \lambda_i^{T}(y) \notin (1 - B_{d_i}^{-T}(y), 1 - B_{d_i}^{+T}(y))$. Therefore $\Lambda^c = <B^c, \lambda^c>$ is an *T*-Internal and *T*-external PNCS in Y.

Correspondingly the subsequent theorems hold for the indeterminate and falsity values.

Theorem 3.3.13 Let Y be a non empty set and $\Lambda = <B, \lambda>$ be a PNCS in Y. If Λ is both *I*-internal and *I*-external, then the complement $\Lambda^c = <B^c, \lambda^c>$ of $\Lambda = <B, \lambda>$ is an *I*-internal and *I*-external PNCS in Y.

Theorem 3.3.14 Let Y be a non empty set and $\Lambda = <B, \lambda>$ be a PNCS in Y. If Λ is both *F*-internal and *F*-external, then the complement $\Lambda^c = <B^c, \lambda^c>$ of $\Lambda = <B, \lambda>$ is an *F*-internal and *F*-external PNCS in Y.

Definition 3.3.15 Let $\Lambda = <B, \lambda> \in P^{\Omega}{}_N$.

If $B_{d_i}^{-T}(y) \le \lambda_i^{T}(y) \le B_{d_i}^{+T}(y)$, $B_{d_i}^{-I}(y) \le \lambda_i^{I}(y) \le B_{d_i}^{+I}(y)$, $\lambda_i^{F}(y) \notin (B_{d_i}^{-F}(y), B_{d_i}^{+F}(y))$

or

$B_{d_i}^{-T}(y) \le \lambda_i^{T}(y) \le B_{d_i}^{+T}(y)$, $B_{d_i}^{-F}(y) \le \lambda_i^{F}(y) \le B_{d_i}^{+F}(y)$, $\lambda_i^{I}(y) \notin (B_{d_i}^{-I}(y), B_{d_i}^{+I}(y))$

or

$B_{d_i}^{-F}(y) \le \lambda_i^{F}(y) \le B_{d_i}^{+F}(y)$, $B_{d_i}^{-I}(y) \le \lambda_i^{I}(y) \le B_{d_i}^{+I}(y)$, $\lambda_i^{T}(y) \notin (B_{d_i}^{-T}(y), B_{d_i}^{+T}(y))$ for all $y \in Y$ .Then Λ is called $\frac{2}{3}$ IPNCS or $\frac{1}{3}$ EPNCS.

Example 3.3.16 The following values of attribute represents the criteria "Types of humans" : Fun loving (the dominant one), Sensitive, Determined, Serious

Contradictory Degree	0	0.50	0.75	1

DOI: 10.5281/zenodo.4030378

Value of attributes	Fun loving	Sensitive	Determined	Serious
Appurtenance Degree $B(y)$	([0.1, 0.6], [0.3, 0.5], [0.4, 0.8])	([0.1, 0.3], [0.2, 0.3], [0.6, 0.8])	([0.1, 0.4] , [0.2, 0. 6], [0.8, 0.9])	([0.3, 0.5], [0.2,0.7], [0.6, 0.8])
$\lambda(y)$	([0.1,0.4, 0.9])	([0.2, 0.5, 0.7])	([0.7, 0.5, 0.8])	([0.3,0.4, 0.3])

Definition 3.3.16 Let $\Lambda = < B, \lambda > \in P^{\Omega}{}_N$.

If $B_{d_i}{}^{-T}(y) \le \lambda_i{}^T(y) \le B_{d_i}{}^{+T}(y)$, $\lambda_i{}^I(y) \notin (B_{d_i}{}^{-I}(y), B_{d_i}{}^{+I}(y))$, $\lambda_i{}^F(y) \notin (B_{d_i}{}^{-F}(y), B_{d_i}{}^{+F}(y))$

or

$B_{d_i}{}^{-F}(y) \le \lambda_i{}^F(y) \le B_{d_i}{}^{+F}(y)$, $\lambda_i{}^T(y) \notin (B_{d_i}{}^{-T}(y), B_{d_i}{}^{+T}(y))$, $\lambda_i{}^I(y) \notin (B_{d_i}{}^{-I}(y), B_{d_i}{}^{+I}(y))$

or

$B_{d_i}{}^{-I}(y) \le \lambda_i{}^I(y) \le B_{d_i}{}^{+I}(y)$, $\lambda_i{}^F(y) \notin (B_{d_i}{}^{-F}(y), B_{d_i}{}^{+F}(y))$, $\lambda_i{}^T(y) \notin (B_{d_i}{}^{-T}(y), B_{d_i}{}^{+T}(y))$ for all $y \in Y$.

Then Λ is called $\frac{1}{3}$ IPNCS or $\frac{2}{3}$ EPNCS.

Example 3.3.17 The following values of attribute represents the criteria " Social Networks " : Whatsapp, Facebook (the dominant one), Instagram, Linkedinn

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Whatsapp	Facebook	Instagram	Linkedin
Appurtenance Degree $B(y)$	([0.2, 0.6], [0.7, 0.5], [0.4, 0.8])	([0.1, 0.3], [0.2, 0.4], [0.6, 0.8])	([0.1, 0.4] , [0.2, 0. 6], [0.8, 0.9])	([0.3, 0.5], [0.2,0.7], [0.6, 0.8])
$\lambda(y)$	([0.3,0.2, 0.9])	([0.4, 0.3, 0.9])	([0.7, 0.2, 0.6])	([0.2, 0.5, 0.4])

Theorem 3.3.14 Let $\Lambda = < B, \lambda > \in P^{\Omega}{}_N$. Then

(i) All IPNCS is a generalisation of ICS.
(ii) All EPNCS is a generalisation of ECS.
(iii) All PNCS is a generalisation of CS.

Proof. Direct proof.

4. Conclusions and future work

In this article,We have introduced the plithogenic fuzzy cubic set, plithogenic intuitionistic fuzzy cubic set, plithogenic neutrosophic cubic sets and their corresponding internal and external cubic sets are defined with examples. Furthermore some of the properties of plithogenic neutrosophic cubic sets are investigated. In the consecutive research, we will study the P-Union, P-Intersection, R-Union, R-Intersection of plithogenic neutrosophic cubic sets.

Funding: "This research received no external funding"

Conflicts of Interest: "The authors declare no conflict of interest."

DOI: 10.5281/zenodo.4030378

References

[1] R.Anita Cruz and F.Nirmala Irudayam, “Neutrosophic Soft Cubic set”, International Journal of Mathematics trends and Technology 46 (2), pp.88-94, 2017.

[2] K.T. Atanassov, “Intuitionistic fuzzy sets”, Fuzzy sets and Systems 20, pp.87- 96,1986.

[3] Y.B.Jun, C.S.Kim and K.O.Yang, “Cubic sets”, Annals of Fuzzy Mathematics and Informatics 4(3),pp.83-98, 2012.

[4] Y.B. Jun, F.Smarandache and C.S. Kim, “Neutrosophic Cubic set”, New Mathematics and Natural Computation, 13 (1), pp.41-54, 2017.

[5] Majid Khan , Ismat Beg and Muhammad Gulistan, “Exponential Laws and Aggregation Operators on Neutrosophic Cubic Sets”, International Journal of Neutrosophic Science (IJNS), Vol. 4, No. 1,pp. 47-71, 2020

[6] Nivetha Martin and Florentin Smarandache, “Plithogenic Cognitive Maps in Decision Making”, International Journal of Neutrosophic Science (IJNS) ,Vol. 9, No. 1,pp. 09-21, 2020

[7] Prem Kumar Singh, “Plithogenic set for multi-variable data analysis”, International Journal of Neutrosophic Science (IJNS), Vol. 1, No. 2, pp. 81-89, 2020

[8] F.Smarandache, “A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic”, Rehoboth: American Research Press, 1999.

[9] F.Smarandache, “Neutrosophic set, a generalisation of the intuitionistic fuzzy sets”, Inteer J. PuBre Appl. Math. 24, pp.287-297,2005.

[10]F.Smarandache, “Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy,NeutrosophyLogic, Set, Probability and Statistics” University of New Mexico,Gallup,NM 87301,USA, 2002.

[11] F.Smarandache “Neutrosophy, A new Branch of Philosophy”,= Multiple Valued Logic /An International Journal, USA, ISSN 1023-6627, Vol;. 8, No.3, pp.297-384, 2002.

[12] F.Smarandache, “Plithogeny, PlithogenicSet, Logic,Probability, and Statistics”, 141 pages, Pons Editions,Brussels,Belgium,2017, arXiv.org (Cornell University), Computer Science-Artificial Intelligence, 03Bxx:abstract:https://arxiv.org/abs/1808.03948,book:https://arxiv.org/ftp/arxiv/papers/1808/1808.03948.pdf,HarvardSAO/NASAADS: http://adsabs.harvard.edu/cgi-bin/bib_query?arXiv:1808.03948.

[13] Turksen, “Interval valued fuzzy sets based on normal forms”, Fuzzy sets and Systems, 20, pp.191-210,1968.

[14] B. Turksen,“ Interval-valued fuzzy sets and compensatory” Fuzzy Sets and Systems, 51, pp.295–307,1992.

[15] I. B. Turksen, “Interval-valued strict preference with Zadeh triples”, Fuzzy Sets and Systems 78, pp.183–195,1996.

[16] Victor Christianto, Robert N. Boyd and Florentin Smarandache “Three Possible Applications of Neutrosophic Logic in Fundamental and Applied Sciences”, International Journal of Neutrosophic Science (IJNS)Vol. 1, No. 2, pp. 90-95, 2020

[17] Wang, H.; Smarandache, F.; Zhang, Y. Q.; Sunderraman, R. “Interval Neutrosophic Sets and logic: Theory and Applications in Computing”; Hexis, Phoenix, Ariz, USA, 2005.

[18] L.A Zadeh, “Fuzzy sets”, Inform and Control 8, pp.338-353,1965.

DOI: 10.5281/zenodo.4030378

Cryptography in Terms of Triangular Neutrosophic Numbers with Real Life Applications

Ali Hamza[1], Muhammad Naveed Jafar *[1], Ahtasham Habib[2], Sara Farooq[2], Ezgi Türkarslan[2]

[1] Department of Mathematics Lahore Garrison University, Sector-C, Av-4, DHA Phase VI 54000 Lahorer, Pakistan
[2] Department of Mathematics Lahore Garrison University, Sector-C, Av-4, DHA Phase VI 54000 Lahorer, Pakistan
[2] Department of Mathematics Lahore Garrison University, Sector-C, Av-4, DHA Phase VI 54000 Lahorer, Pakistan
[2] Department of Mathematics Lahore Garrison University, Sector-C, Av-4, DHA Phase VI 54000 Lahorer, Pakistan
[2]Department of Mathematics, TED University, Ön Cebeci, Ziya Gökalp Cd. 48/A, 06420 Çankaya/Ankara, Turkey

* Correspondence: naveedjafar@lgu.edu.pk

Abstract

In this article, our main focus is to put forward the concept of Cryptography in terms of triangular neutrosophic numbers. This kind of cryptography is really reliable, manual, secure, and based on few simple steps. All the encryption and decryption are easy to proceed (mention below). As we know, Public-key cryptography as an indefatigable defender for human privacy and use as information transfer from the ages. various concepts are available with regard to cryptography e.g. Elliptic curve cryptography. TNNC (Triangular neutrosophic numbers cryptography) is familiar with basic concepts of math as well as applicable in different situations e.g. code cryptography, detailed view cryptography, and Graph cryptography encryption facilitate.

Keywords: Cryptography, Triangular Neutrosphic numbers, Code Encryption, Detailed overview encryption.

Why is Cryptography in Terms of Triangular neutrosophic numbers required?

- Easy to use and no computer work required to proceed (Fully manual).
- With the help of math's law, it is impossible to decrypt without a key.
- No internet required for that kind of cryptography.
- Convenient for personal use as well as for personal records.
- Based on basic math calculation and possible to proceed with one formula.

1 Introduction and History

Analysts, investigators, and mathematicians from all over the world introduce a wide range of techniques to solve decision-making problems. After great approaches in the field of medicine, selection problem, human resources, and now with the help of Triangular neutrosophic numbers TNNs, new possibilities are available, with respect to cryptography. Researchers give their best, as fundamental of cryptography, principles, applications, and encryption slandered [1-2]. Cryptography has become really handy and image cryptography [3-6], color image cryptography has been introduced [7] and chocolate cryptography also has been developed [8]. Stegano-cryptosystems introduced as well as researchers have also create new approaches in cryptography including S-Boxes [9-19].

DOI: 10.5281/zenodo.4127138
Received: Jun 04, 2020 Accepted: September 14, 2020

Life is full of uncertainty and decision making, and math is a key to solve daily life problems, on the way to serve humankind. Fuzzy logic has its own importance (introduced by Molodtsov) [20] in 1999 to deal with uncertainty. After the fuzzy set theory [21-23] Intuitionistic fuzzy sets was introduced [24] then neutrosophic sets [25]. Fuzzy and Neutrosophic set theory plays an important role in the field of uncertainty and to handle situation regards to multi-criteria decision making and in the line of evolution, neutrosophic fuzzy numbers [26] and finally Triangular neutrosophic numbers were introduced [27].

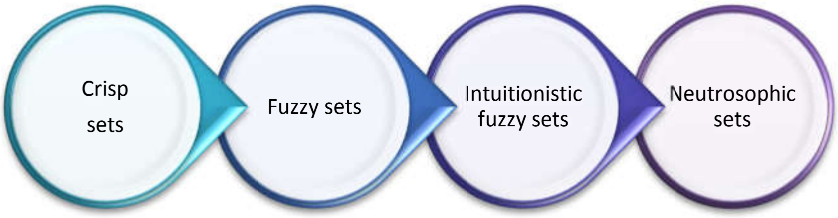


Figure 1

Researchers also exaggerate the concept of neutrosophic [28-40]. Neutrosophic sets, which we use, are base on truth, indeterminacy, and falsity. Wang [41] introduces Single-value neutrosophic sets and Ye [42] further furnished the concept of neutrosophic sets. Smarandache and many other researchers [43-50] introduce TOPSIS technique and their services in multi-criteria decision-making (MCDM), also appreciate able. As well as many other researchers from different origins of the world introduce many possibilities and put forward the concept of Multi-Criteria decision-making (MCDM) [51-57] and the properties of triangular and pentagonal neutrosophic number are proposed by A. Chakraborty [58-59].

UP until the 1970s, Encryption all over the world was based on symmetric ciphers, after that technology rises and overcomes Manual Cryptography methods. But remember that, online cryptography is risky and complex in some situations. Hence by TNNC Manual cryptography is simple to use secure, undemanding and Cipher Text is now more complex and impossible to break. With huge applications in code encryption, name encryption and complete information of the company is now secure manually.

1.1 Motivation and Benefits

As we all know that, neutrosophic is advance universe and really convenient to solve MCDM problems and now introduces new possibilities in Cryptography. With the help of Triangular Neutrosophic concept manual cryptography resurrect as well as, with more security, less demanding and easy steps to occur. This concept is totally new, Cipher text is now more complex, deal with truth, indeterminacy and falsity at the same time, Graph Encryption possible and with one key this cryptography is now personal cryptography. Hence, all the personal records (Account number, budget, earning, Persons for different situations, Graphs) are all Encrypted manually.

1.2 The Paper Presentation

In this current epoch, the concept of manual Cryptography as TNNC is extended.

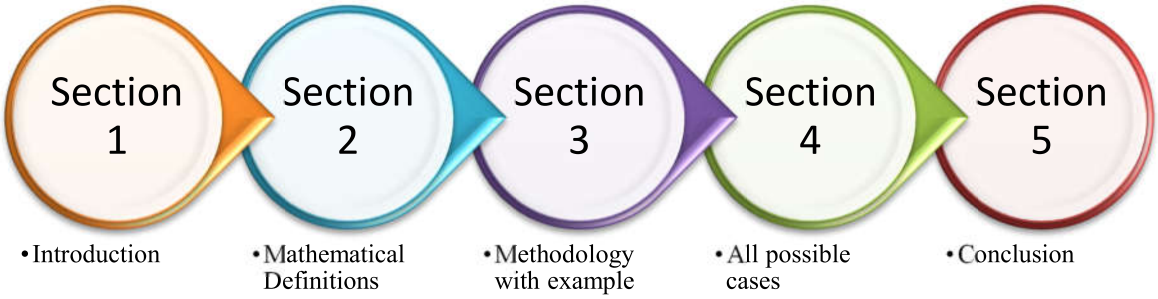
- Formation of Triangular Neutrosophic numbers Cryptography (TNNC) with detailed algorithm.
- Code Encryption as well as complete bank accounts encryption with budget amount.

DOI: 10.5281/zenodo.4127138

- Name of Persons for different situation also encrypted.
- Complete company data encrypted manually.

1.3 Structure of Paper

The article depends upon these following prospective.



2 Mathematical Definitions

Definition 2.1: Neutrosophic set: Set $\breve{nA}$ is neutrosophic if $\breve{nA} = \{\langle \tilde{x}; [\tilde{T}_{\breve{nA}}(\tilde{x}), \tilde{I}_{\breve{nA}}(\tilde{x}), \tilde{F}_{\breve{nA}}(\tilde{x})]\rangle : \tilde{x} \in \tilde{X}\}$, where $\tilde{T}_{\breve{nA}}(\tilde{x}) \mapsto [0,1]$, $\tilde{I}_{\breve{nA}}(\tilde{x})$ *and* $\tilde{F}_{\breve{nA}}(\tilde{x})$ are membership functions of truth, indeterminacy and falsity with this following relation:

$$\mathbf{0^-} \leq \tilde{T}_{\breve{nA}}(\tilde{x}), \tilde{I}_{\breve{nA}}(\tilde{x}), \tilde{F}_{\breve{nA}}(\tilde{x}) \leq \mathbf{3^+}$$

Definition 2.2: Triangular Neutrosophic Numbers: Single-value Triangular neutrosophic number is given as $\tilde{A}_{\overline{Neu}} = (\tilde{p}_1, \tilde{p}_2, \tilde{p}_3 : \tilde{r}_1, \tilde{r}_2, \tilde{r}_3)$ and their truth, indeterminacy and falsity membership is given as:

$$\tilde{T}_{\tilde{A}_{\overline{Neu}}}(\tilde{x}) = \begin{cases} \dfrac{\tilde{x} - \tilde{p}_1}{\tilde{p}_1 - \tilde{p}_2} & for\ \tilde{p}_1 \leq x < \tilde{p}_2 \\ 1 & when \quad \tilde{x} = \tilde{p}_2 \\ \dfrac{\tilde{p}_3 - \tilde{x}}{\tilde{p}_3 - \tilde{p}_2} & for\ \tilde{p}_2 < x \leq \tilde{p}_3 \\ 0 & \quad otherwise \end{cases}$$

$$\tilde{I}_{\tilde{A}_{\overline{Neu}}}(\tilde{x}) = \begin{cases} \dfrac{\tilde{q}_2 - \tilde{x}}{\tilde{q}_2 - \tilde{q}_1} & for\ \tilde{q}_1 \leq x < \tilde{q}_2 \\ 0 & when \quad \tilde{x} = \tilde{q}_2 \\ \dfrac{\tilde{x} - \tilde{q}_2}{\tilde{q}_3 - \tilde{q}_2} & for\ \tilde{q}_2 < x \leq \tilde{q}_3 \\ 1 & \quad otherwise \end{cases}$$

DOI: 10.5281/zenodo.4127138

$$\tilde{F}_{\tilde{A}_{\widetilde{Neu}}}(\tilde{x}) = \begin{cases} \dfrac{\tilde{x}-\tilde{p}_1}{\tilde{p}_1-\tilde{p}_2} & for\ \tilde{p}_1 \le x < \tilde{p}_2 \\ 1 & when \quad \tilde{x} = \tilde{p}_2 \\ \dfrac{\tilde{p}_3-\tilde{x}}{\tilde{p}_3-\tilde{p}_2} & for\ \tilde{p}_2 < x \le \tilde{p}_3 \\ 0 & otherwise \end{cases}$$

Where $0 \le \tilde{T}_{\tilde{A}_{\widetilde{Neu}}}(\tilde{x}) + \tilde{I}_{\tilde{A}_{\widetilde{Neu}}}(\tilde{x}) + \tilde{F}_{\tilde{A}_{\widetilde{Neu}}}(\tilde{x}) \le 3, \tilde{x} \in \tilde{A}_{\widetilde{Neu}}$

Parametric form is defined as: $(\tilde{A}_{\widetilde{Neu}})_{\tilde{\alpha},\tilde{\beta},\tilde{\gamma}}$=$[\tilde{T}_{\widetilde{Neu}1}(\tilde{\alpha}), \tilde{T}_{\widetilde{Neu}2}(\tilde{\alpha}) : \tilde{I}_{\widetilde{Neu}1}(\tilde{\beta}), \tilde{I}_{\widetilde{Neu}2}(\tilde{\beta}) : \tilde{F}_{\widetilde{Neu}1}(\tilde{\gamma}),\ \tilde{F}_{\widetilde{Neu}2}(\tilde{\gamma})]$, where,

$\tilde{T}_{\widetilde{Neu}1}(\tilde{\alpha})$=$\tilde{p}_1 + \tilde{\alpha}(\tilde{p}_2 - \tilde{p}_1)$, $\tilde{T}_{\widetilde{Neu}2}(\tilde{\alpha})$=$\tilde{p}_3 - \tilde{\alpha}(\tilde{p}_3 - \tilde{p}_2)$, $\tilde{I}_{\widetilde{Neu}1}(\tilde{\beta})$=$\tilde{q}_2 - \tilde{\beta}(\tilde{q}_2 - \tilde{q}_1)$, $\tilde{I}_{\widetilde{Neu}2}(\tilde{\beta})$=$\tilde{q}_2 + \tilde{\beta}(\tilde{q}_3 - \tilde{q}_2)$, $\tilde{F}_{\widetilde{Neu}1}(\tilde{\gamma})$=$\tilde{r}_2 - \tilde{\gamma}(\tilde{r}_2 - \tilde{r}_1)$, $\tilde{F}_{\widetilde{Neu}2}(\tilde{\gamma})$=$\tilde{r}_2 + \tilde{\gamma}(\tilde{r}_3 - \tilde{r}_2)$, here $0<\tilde{\alpha} \le 1$, $0<\tilde{\beta} \le 1, 0 < \tilde{\gamma} \le 1$ and $0 < \tilde{\alpha} + \tilde{\beta} + \tilde{\gamma} \le 3$.

Definition 2.3: Group is algebraic structure formed by:

¤ Set with element G.

¤ Closed operation on set G, as well as associative. Which is $(\tilde{a}.\tilde{b}).\tilde{c} = \tilde{a}.(\tilde{b}.\tilde{c})$, for $\tilde{a},\tilde{b},\tilde{c} \in G$.

¤ Identity element

¤ Availability of inverse under set operation

Group which have set operation commutative ($\tilde{a}.\tilde{b} = \tilde{b}.\tilde{a}$) defined as *abelian group.*

Here we have ''+'' as set operation and ''0'' as identity element. We gain a valid group by these prospective and by set of integers ''Z'', where pain of inverse and integers with opposite signs are used. With no inverse natural number ''N'' does not form a group.

Definition 2.4: With the operation (.) and $\tilde{a}$ be a element of group G with identity 1. Order of $\tilde{a}$ and it is of smallest integer $\tilde{n}$ such that:

$$\underbrace{\tilde{a}.\tilde{a}.\tilde{a}.\tilde{a}\ldots\tilde{a}}_{\tilde{n}} = 1$$

Set $\{\tilde{a}, \tilde{a}^2, \tilde{a}^3, \tilde{a}^4, \ldots, \tilde{a}^{\tilde{n}}\}$ compose a cyclic subgroup of G with order $\tilde{n}$, here $\tilde{a}$ is g*enerator* for the subgroup.

3 Methodology with Example

Suppose the number in 0.97 as plain Text and the key for encryption is 0.21. The procedure depends upon following steps:

We have three values in Triangular fuzzy numbers.

Step 1

Add the key in second value of set:

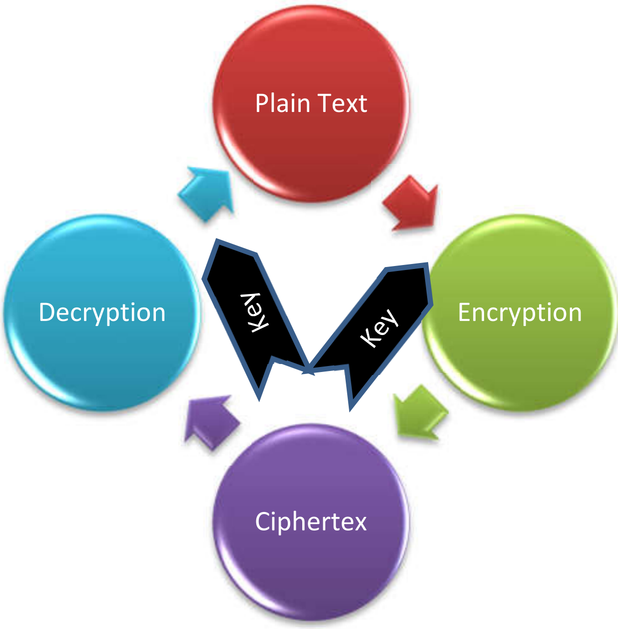
{0.97,0.97,0.97} ⟼ add key (0.21) in second number ⟼ {0.97,1.18,0.97}

Step 2

Apply formula $\frac{\tilde{a}_1+\tilde{b}_1+\tilde{c}_1}{3}$ for the process of encryption. The value after applying formula on above set is 1.04. In this case 1.04 is Cipher text.

Step 3 (Decryption)

For the process of Decryption, firstly multiply the Cipher text by 3 (Result 3.12), then subtract the key (Result 2.91) and finally divide the result by 3. Hence by this process we will get the plain text 0.97.



Security Analysis And Necessary Information

When we apply the key (methodology) then after applying the formula, it is impossible to move back to secure values without a key.

It's all up to you, which key you want to suppose. It's just like a simple pin code but all the encryption and decryption is only possible with pin code (key), which is unable to guess or break.

4 Case study

Few cases are given below, which mention the requirement of TNNC and show how the process proceeds with a few simple steps, key and done as maximum security.

4.1 Case 1: Code Cryptography

In this current era, cryptography is really familiar for privacy. Suppose the deal $\tilde{A}$ of a company depend on three different values: Account number as Truth. Range of amount for the deal depend upon Indeterminacy and Falsity. Put these values after zero.

Truth Indeterminacy Falsity

$\tilde{A}$ ={(0.9878677721, 0.9878677721, 0.9878677721) (0.9700, 0.9700, 0.9700) (0.10999, 0.10999, 0.10999)}

Let Key = 0.9786

Encryption: Add the key in the middle value of all above:

$\tilde{A}$ ={(0.9878677721, 1.966467772, 0.9878677721) (0.9700, 1.9486, 0.9700) (0.10999, 1.08859, 0.10999)}

Apply Formula $\frac{\tilde{a}_1+\tilde{b}_1+\tilde{c}_1}{3}$ and get the result as:

$\tilde{A}$ ={(1.31406777207) (1.2962) (0.43619)} This is the **Cipher Text.**

Decryption: Multiply the Cipher text by 3:

$\tilde{A}$ ={(3.94220331621) (3.8886) (1.30857)}, Now Subtract the key:

$\tilde{A}$ ={(2.96360331621) (2.91) (2.32997)}, Now divide it by 3:

Truth Indeterminacy Falsity

$\tilde{A}$ ={(0.9878677721, 0.9878677721, 0.9878677721) (0.9700, 0.9700, 0.9700) (0.10999, 0.10999, 0.10999)}

Table for Case 2:

This table is designed to convert the alphabets of names in fuzzy values and this process is required because security is the main motto of this research article. This table is unique and specially designed by the author and the table is based on simple values, which are supposed. Once we convert the unique alphabets of names in fuzzy values, then we will be good to go and able to apply it on a few simple steps of triangular neutrosophic numbers.

$\breve{A}$ 0.01	$\breve{B}$ 0.02	$\breve{C}$ 0.03	$\breve{D}$ 0.04	$\breve{E}$ 0.05	$\breve{F}$ 0.06	$\breve{G}$ 0.07
$\breve{H}$ 0.08	$\breve{I}$ 0.09	$\breve{J}$ 0.10	$\breve{K}$ 0.11	$\breve{L}$ 0.12	$\breve{M}$ 0.13	$\breve{N}$ 0.14
$\breve{O}$ 0.15	$\breve{P}$ 0.16	$\breve{Q}$ 0.17	$\breve{R}$ 0.18	$\breve{S}$ 0.19	$\breve{T}$ 2.0	$\breve{U}$ 2.1
$\breve{V}$ 2.2	$\breve{W}$ 2.3	$\breve{X}$ 2.4	$\breve{Y}$ 2.5	$\breve{Z}$ 2.6		

4.2 Case 2: Communication Cryptography

Suppose a company is suffering from quick base action and decided to mention volunteer to deal with that critical situation. Two senior persons of company are communicating and the topic under discussion is ideal person, backup person and really not recommended person to deal with that situation.

DOI: 10.5281/zenodo.4127138

The three persons are William as ideal person (Truth), Mason as backup (Indeterminacy) and john not recommended for this situation (Falsity). Now convert the names in the form of numbers.

This methodology, consist of one more step: Convert the name into number. For this procedure, Convert the name into less letter. William $\longmapsto \breve{W}\breve{L}\breve{M}$, Mason $\longmapsto \breve{M}\breve{S}\breve{N}$ and John $\longmapsto \breve{J}\breve{H}\breve{N}$. Know by the help of above table entertain them with numerical values. $\breve{W}\breve{L}\breve{M} \longmapsto$ 2.3+ 0.12+ 0.13=2.55. $\breve{M}\breve{S}\breve{N} \longmapsto$ 0.13+ 0.19+ 0.14=0.46 and $\breve{J}\breve{H}\breve{N} \longmapsto$ 0.10+ 0.08+ 0.14=0.32.

Truth Indeterminacy Falsity

$$\tilde{A} = \{(2.55, 2.55, 2.55) \quad (0.46, 0.46, 0.46) \quad (0.32, 0.32, 0.32)\}$$

Let Key = 0.77

Encryption: Add the key in the middle value of all above:

$$\tilde{A} = \{(2.55, 3.32, 2.55) \quad (0.46, 1.23, 0.46) \quad (0.32, 1.09, 0.32)\}$$

Apply Formula $\frac{\tilde{a}_1+\tilde{b}_1+\tilde{c}_1}{3}$ and get the result as:

$\tilde{A} = \{(2.806666667) \ (0.7166666667) \ (0.5766666667)\}$ This is the **Cipher Text.**

Decryption: Multiply the Cipher text by 3:

$\tilde{A} = \{(8.42) \ (2.15) \ (1.73)\}$, Now Subtract the key:

$\tilde{A} = \{(7.65) \ (1.38) \ (0.96)\}$, Now divide it by 3:

Truth Indeterminacy Falsity

$$\tilde{A} = \{(2.55,2.55,2.55) \quad (0.46,0.46,0.46) \quad (0.32,0.32,0.32)\}$$

Finally compare the number with name, by the help of table as $\breve{W}\breve{L}\breve{M} \longmapsto$ 2.55, $\breve{M}\breve{S}\breve{N} \longmapsto$ 0.46 and $\breve{J}\breve{H}\breve{N} \longmapsto$ 0.32.

4.3 Case 3: Graph Cryptography

Suppose a person want to be aware of online scam and want to save his personal company information manually. Here, with the help of one single key he can store multiple data manually with few single steps.

a) Progress per 10 days

The progress is 89%, 29% and 69%, on the base of 1000$. Add zero in front of numbers.

$$\tilde{A} = \{(0.89, 0.89, 0.89) \quad (0.29, 0.29, 0.29) \quad (0.69, 0.69, 0.69)\}$$

Let Key = 0.77

Encryption: Add the key in the middle value of all above:

$$\tilde{A} = \{(0.89, 1.66, 0.89) \quad (0.29, 1.06, 0.29) \quad (0.69, 1.46, 0.69)\}$$

Apply Formula $\frac{\tilde{a}_1+\tilde{b}_1+\tilde{c}_1}{3}$ and get the result as:

DOI: 10.5281/zenodo.4127138

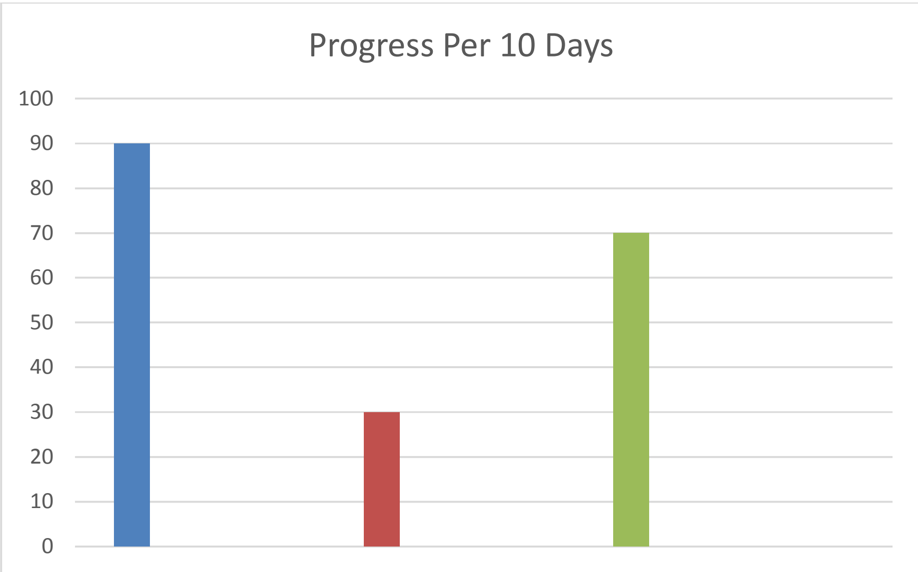
$\tilde{A}$ ={(1.146666667) (0.5466666667) (0.9466666667)} This is the **Cipher Text.**

Decryption: Multiply the Cipher text by 3:

$\tilde{A}$ ={(3.44) (1.64) (2.84)}, Now Subtract the key:

$\tilde{A}$ ={(2.67) (0.87) (2.07)}, Now divide it by 3:

$\tilde{A}$ ={(0.89, 1.66, 0.89) (0.29, 1.06, 0.29) (0.69, 1.46, 0.69)}



b) Budget

Now the most personal thing, need to hide is company budget, for this purpose we just have to follow these following steps. Firstly, convert it in percentage and then encrypt and decrypt it. The percentages are 39% as remaining, 42% and 19% are investment per 15 days.

$\tilde{A}$ ={(0.39, 0.39, 0.39) (0.42, 0.42, 0.42) (0.19, 0.19, 0.19)}

Let Key = 0.77

Encryption: Add the key in the middle value of all above:

$\tilde{A}$ ={(0.39, 1.16, 0.39) (0.42, 1.19, 0.42) (0.19, 0.96, 0.19)}

Apply Formula $\frac{\tilde{a}_1+\tilde{b}_1+\tilde{c}_1}{3}$ and get the result as:

$\tilde{A}$ ={(0.6466666667) (0.6766666667) (0.4466666667)} This is the **Cipher Text.**

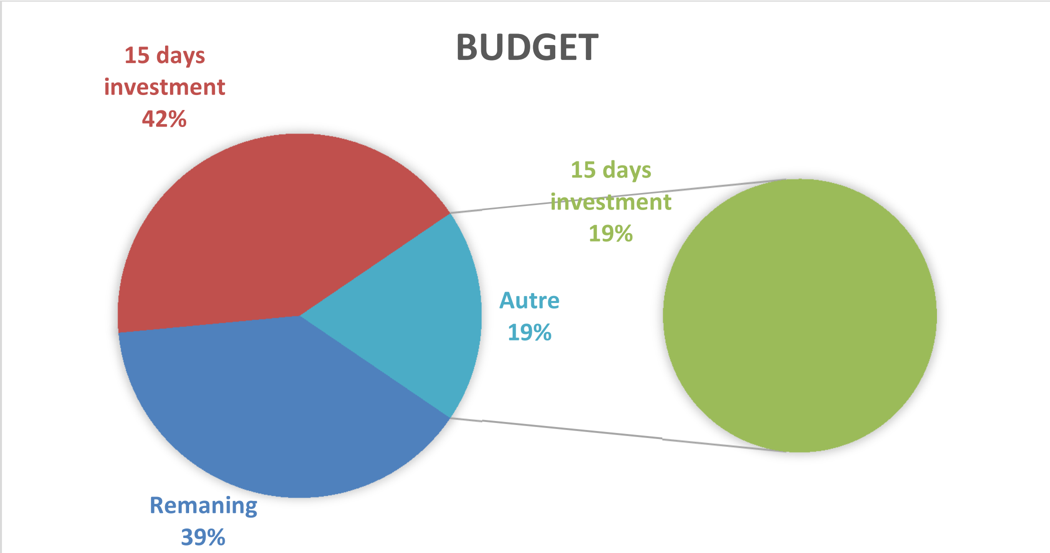
Decryption: Multiply the Cipher text by 3:

$\tilde{A}$ ={(1.94) (2.03) (1.34)}, Now Subtract the key:

$\tilde{A}$ ={(1.17) (1.26) (0.57)}, Now divide it by 3:

DOI: 10.5281/zenodo.4127138

$$\tilde{A} = \{(0.39, 0.39, 0.39) \quad (0.42, 0.42, 0.42) \quad (0.19, 0.19, 0.19)\}$$



c) Earning per 10 days

The progress is 899$, 299$ and 699$

$$\tilde{A} = \{(0.899, 0.899, 0.899) \quad (0.299, 0.299, 0.299) \quad (0.699, 0.699, 0.699)\}$$

Let Key = 0.77

Encryption: Add the key in the middle value of all above:

$$\tilde{A} = \{(0.899, 1.669, 0.899) \quad (0.299, 1.069, 0.299) \quad (0.699, 1.469, 0.699)\}$$

Apply Formula $\frac{\tilde{a}_1+\tilde{b}_1+\tilde{c}_1}{3}$ and get the result as:

$\tilde{A} = \{(1.155666667)\ (0.5556666667)\ (0.9556666667)\}$ This is the **Cipher Text.**

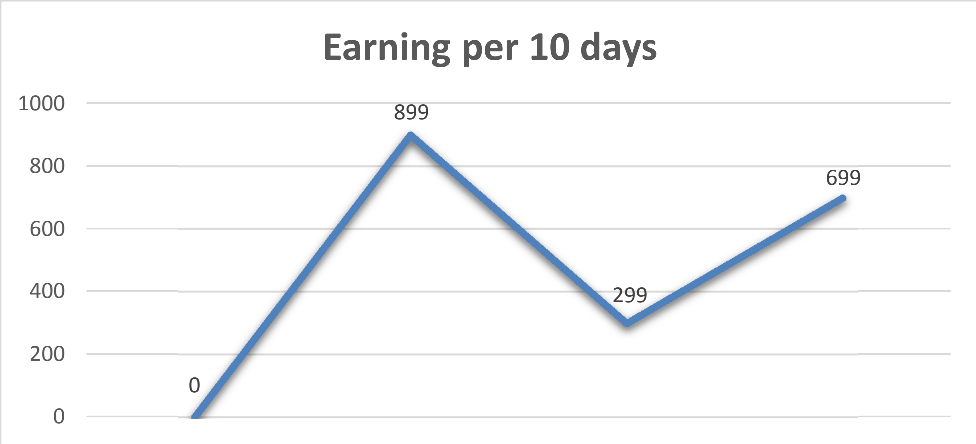
Decryption: Multiply the Cipher text by 3:

$\tilde{A} = \{(3.467)\ (1.667)\ (2.867)\}$, Now Subtract the key:

$\tilde{A} = \{(2.697)\ (0.897)\ (2.097)\}$, Now divide it by 3:

$$\tilde{A} = \{(0.899, 0.899, 0.899) \quad (0.299, 0.299, 0.299) \quad (0.699, 0.699, 0.699)\}$$

DOI: 10.5281/zenodo.4127138



Hence, by the help of TNNC, the complete data of company encrypted including progress, budget and earning as well as data depend upon three different parts.

Limitations

- All the calculations are basic and manual, hence image cryptography and sentence encryption are not possible yet.
- All the calculations are in regards to Triangular netrosophic numbers, so all the codes and values during the process lie between 0 and 3.
- Without the internet but with the help of reliable software like MATLAB, that kind of cryptography is possible and offline apps also developed for this procedure.

5 Conclusion

In this article, our main focus is to promote manual cryptography, which is more secure, less demanding and it is possible to create a personal record with a single key and by least step. All the necessary requirements and limitations of TNNC are also mentioned. Triangular neutrosophic numbers can be used in manual cryptography and now it is possible to create personal record secure manually with a single key. Cypher text is more secure, complex, and impossible to break without a key. The complete procedure of this new concept is available, and all possible personal records in regard to business or for any other condition now encrypted with few simple steps.

Acknowledgement: The authors are fully thankful to the respected Editor-in-chief as well as really great-full to their valuable comments and suggestions, which make this research better and reliable.

References

[1]. Delfs, H.and Knebl, H. Introduction to Cryptography Principles and Applications, *Springer*,2007.

[2] Daemen.J and V. Rijmen, The Design of RIJNDAEL: AES - The Advanced Encryption Standard, *Springer*, Berlin, Germany, 2002.

[3] H. Al-Assam, R. Rashid, and S. Jassim, "Combining steganography and biometric cryptosystems for secure mutual authentication and key exchange, "*in Proceedings of the 2013 8th International Conference for Internet Technology and Secured Transactions (ICITST '13)*, pp. 369–374, IEEE, London, UK, March 2013.

[4] A. Philip, "A generalized pseudo-Knight's tour algorithm for encryption of an image,"*IEEE Potentials,* vol.32,no.6,pp.10–16, 2013.

[5] D. Bloisi and L. Iocchi, "Image based steganography and cryptography, "*in Proceedings of the 2nd International Conference on Computer Vision Theory and Applications''* (VISAPP'07), pp.127–134,Barcelona,Spain,March2007.

[6] H. Sharma, "Secure image hi dingal gorithm using cryptography and steganography, "*IOSR Journal of Computer Engineering*, vol. 13,no.5,pp.1–6,2013.

[7] J. Liu, H. Jin, L. Ma, Y. Li, and W. Jin, "Optical color image encryption based on computer generated hologram and chaotic theory, "Optics Communications,vol.307,pp.76–79,2013. [3] C. E. Shannon, "Communication theory of secrecy systems," *Bell System Technical Journal*, vol.28,no.4,pp.656–715,1949.

[8]. Bachman, D. J.; Brown, E. A.; Norton, A.H, Chocolate Key Cryptography, *Mathematics Teacher*, 104(2),2010, p100-104.

[9] J. Liu, B. Wei, X. Cheng, and X. Wang, "An AESS-box to increase complexity and cryptographic analysis," *in Proceedings of the 19th International Conference on Advanced Information Networking and Applications (AINA '05),* pp. 724–728, Taipei, Taiwan, March2005.

[10] J. Rosenthal, "A polynomial description of the Rijndael advanced encryption standard*," Journal of Algebra and Its Applications*, vol.2,no.2,pp.223–236,2003.

[11]. Koç, Ç.K, Cryptographic Engineering, *Springer*, 2009, PP 125-128.

[12] K. Challita and H. Farhat, "Combining steganography and cryptography: new directions," *International Journal of New Computer Architectures and their Applications*, vol. 1, no. 1, pp. 199–208,2011.

[13] N. Ferguson, R. Schroeppel, and D. Whiting, "A simple algebraic representation of Rijndael," *in Selected Areas in Cryptography SAC'01, vol.2259 of Lecture Notesin Computer Science,* pp.103– 111, Springer, 2001.

[14] S. Murphy and M. J. Robshaw, "Essential algebraic structure within the AES," *in Advances in Cryptology—CRYPTO 2002: 22nd Annual International Cryptology Conference Santa Barbara, California, USA, August18–22,2002 Proceedings, vol.2442 of Lecture Notesin Computer Science*, pp.1–16,Springer,Berlin, Germany,2002.

[15] L. Cui and Y. Cao, "A new S-box structure named affine-power affine, "*International Journal of Innovative Computing, Information and Control*, vol.3, no.3, pp.751–759,2007.

[16] L. Jingmei, W. Baodian, and W. Xinmei, "One AES S-box to increase complexity and its cryptanalysis*," Journal of Systems Engineering and Electronics*, vol.18, no.2, pp.427–433,2007.

[17] M. Khan and N.A. Azam, "Right translated AES gray S-boxes," *Security and Communication Networks*, vol. 8, no. 9, pp. 1627– 1635,2015.

[18] M. Khan and N.A. Azam, "S-boxes based on affine mapping and orbitofpowerfunction,"*3D Research*, vol.6, article12,2015.

[19] M.-T. Tran, D.-K. Bui, and A.-D. Duong, "Gray S-box for advanced encryption standard*," in Proceedings of the International Conference on Computational Intelligence and Security (CIS'08)*, pp.253–258, December2008.

[20] Molodtsov, D.(1999). *Soft set theory - First results, Computers and mathematics with applications*. 37, 19-31.

[21] Zadeh, L.A. Fuzzy sets. *Inf. Control* **1965**, *8*, 338–353.

[22] Chang, S.S.L.; Zadeh, L.A. On fuzzy mappings and control. *IEEE Trans. Syst. Man Cybern.* **1972**, *2*, 30–34.

[23] Dubois, D.; Prade, H. Operations on fuzzy numbers. *Int. J. Syst. Sci.* **1978**, *9*, 613–626.

[24] Atanassov, K.T. *Intuitionistic Fuzzy Sets*; VII ITKR's Session: Sofia, Bulgarian, 1983.

[25] Smarandache, F. *A Unifying Field in Logics Neutrosophy: Neutrosophic Probability*; American Research Press: Rehoboth, DE, USA, 1998.

[26] F. Smarandache, Neutrosophic set, a generalization of the intuitionistic fuzzy sets, Inter. J. Pure Appl. Math. Vol.24 , pp. 287–297, 2005.

[27] Chakraborty, A.; Mondal, S. P.; Ahmadian, A.; Senu, N.; Alam, S.; and Salahshour, S.; Different Forms of Triangular Neutrosophic Numbers, De-Neutrosophication Techniques, and their Applications, Symmetry, vol 10, 327, 2018. ; doi:10.3390/sym10080327.

[28] Abdel-Basset, M., Atef, A., & Smarandache, F. A hybrid Neutrosophic multiple criteria group decision making approach for project selection. *Cognitive Systems Research*, vol 57, pp. 216-227, 2019.

[29] Abdel-Baset, M., Chang, V., & Gamal, A. Evaluation of the green supply chain management practices: *A novel neutrosophic approach. Computers in Industry*, vol 108, pp. 210-220, 2019.

[30] Abdel-Baset, M., Chang, V., Gamal, A., & Smarandache, F. An integrated neutrosophic ANP and VIKOR method for achieving sustainable supplier selection: *A case study in the importing field. Computers in Industry*, vol 106, pp. 94-110, 2019.

[31] Abdel-Basset, Mohamed, Mumtaz Ali, and Asma Atef. "Resource levelling problem in construction projects under neutrosophic environment." *The Journal of Supercomputing*, pp.1-25, 2019.

[32] Abdel-Basset, M., Manogaran, G., Gamal, A., & Smarandache, F. A group decision making framework based on neutrosophic TOPSIS approach for smart medical device selection. *Journal of medical systems*, vol 43(2), pp. 38, 2019.

[33] Abdel-Basset, M., Manogaran, G., Gamal, A., & Smarandache, F. A group decision-making framework based on the neutrosophic TOPSIS approach for smart medical device selection. *Journal of medical systems*, vol 43, issue 2, pp. 38-43, 2019.

[34] Abdel-Basset, M., Manogaran, G., Gamal, A., & Smarandache, F. A hybrid approach of neutrosophic sets and DEMATEL method for developing supplier selection criteria. *Design Automation for Embedded Systems*, pp.1-22, 2018.

[35] Abdel-Basset, M., Saleh, M., Gamal, A., & Smarandache, F. An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. *Applied Soft Computing*, vol 77, pp. 438-452, 2019.

[36] Saqlain, M., Jafar, M. N., Riaz, M. "A New Approach of Neutrosophic Soft Set with Generalized Fuzzy TOPSIS in Application of Smart Phone Selection," *Neutrosophic Sets and Systems*, vol. 32, pp. 307-316, 2020. DOI: 10.5281/zenodo.3723161.

DOI: 10.5281/zenodo.4127138

[37] Saqlain M, Jafar N, Hamid R, Shahzad A. "Prediction of Cricket World Cup 2019 by TOPSIS Technique of MCDM-A Mathematical Analysis*," International Journal of Scientific & Engineering Research*, vol 10(2), pp. 789-792, 2019.

[38] Saqlain. M, Jafar. N. M, and Muniba. K, "Change in The Layers of Earth in Term of Fractional Derivative: *A Study," Gomal University Journal of Research*, vol 34(2), pp. 1-13, 2018.

[39] Saqlain. M., Jafar. N. and Riffat. A., "Smart phone selection by consumers' in Pakistan: FMCGDM fuzzy multiple criteria group decision making approach," *Gomal University Journal of Research*, vol 34(1), pp. 27-31, 2018.

[40] Saqlain M, Saeed M, Ahmad M. R, Smarandache F, Generalization of TOPSIS for Neutrosophic Hypersoft set using Accuracy Function and its Application, *Neutrosophic Sets and Systems (NSS),* vol 27, pp. 131137, 2019.

[41] Wang, H.B.; Smarandache, F.; Zhang, Y.Q.; Sunderraman, R. Single Valued Neutrosophic Sets. Tech. Sci. Appl. Math. 2010. Available online: http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.309.9470&re p=rep1&type=pdf (accessed on 31 July 2018).

[42] Ye, J. A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. J. Int. Fuzzy Syst., vol 26,pp. 2459–2466, 2014.

[43] P. Biswas, S. Pramanik, and B. C. Giri. "A new methodology for neutrosophic multi-attribute decision making with unknown weight information," *Neutrosophic Sets and Systems*, vol 3, pp. 42-52, 2014.

[44] Parimala ,M,. Karthika, M, Florentin Smarandache , Said Broumi, "On αω-closed sets and its connectedness in terms of neutrosophic topological spaces," *International Journal of Neutrosophic Science*, Volume 2 , Issue 2, pp. 82-88 , 2020.

[45] Riaz. M., Saqlain. M. and Saeed. M., "Application of Generalized Fuzzy TOPSIS in Decision Making for Neutrosophic Soft set to Predict the Champion of FIFA 2018: *A Mathematical Analysis," Punjab University Journal of Mathematics*, vol 51(8), pp.111-126, 2019.

[46] Riaz. M., Saeed. M. Saqlain. M. and Jafar. N, "Impact of Water Hardness in Instinctive Laundry System based on Fuzzy Logic Controller," *Punjab University Journal of Mathematics*, vol 51(4), pp. 73-84, 2018.

[47] Smarandache, F." Neutrosophy. Neutrosophic probability, set, and logic, ProQuest Information & Learning, *Ann Arbor, Michigan*, USA, 1998.

[48] Smarandache, F., Pramanik, S., "New Neutrosophic Sets via Neutrosophic Topological Spaces," *In Neutrosophic Operational Research; Eds.; Pons Editions: Brussels, Belgium*, vol I, pp. 189–209, 2017.

[49] S. Pramanik, P. P. Dey and B. C. Giri, "TOPSIS for single valued neutrosophic soft expert set based multiattribute decision making problems," *Neutrosophic Sets and Systems,* vol 10, pp. 88-95, 2015.

[50] T. Bera and N. K. Mahapatra, Introduction to neutrosophic soft groups, *Neutrosophic Sets and Systems*, vol 13, pp. 118-127, 2016. doi.org/10.5281/zenodo.570845.

[51] Chakraborty, A. "A New Score Function of Pentagonal Neutrosophic Number and its Application in Networking Problem," *International Journal of Neutrosophic Science*, Volume 1, Issue 1, pp. 40-51, 2020.

DOI: 10.5281/zenodo.4127138

[52] Deli I, Broumi S. Neutrosophic soft matrices and NSM decision making, *Journal of Intelligent and Fuzzy System* vol 28, pp.2233–2241, 2015.

[53] Edalatpanah, S. A., "A Direct Model for Triangular Neutrosophic Linear Programming," *International Journal of Neutrosophic Science*, Volume 1, Issue 1, pp. 19-28, 2020.

[54] F. Smarandache, Neutrosophic set, a generalization of the intuitionistic fuzzy sets, *Inter. J. Pure Appl. Math*. Vol.24, pp. 287–297, 2005.

[55] I. Deli and S. Broumi, "Neutrosophic Soft Matrices and NSM-decision Making," *Journal of Intelligent and Fuzzy Systems*, vol 28(5), pp. 2233-2241, 2015.

[56] K. Mondal, and S. Pramanik. Neutrosophic decision making model of school choice. *Neutrosophic Sets and Systems*, vol 7, pp. 62-68, 2015.

[57] Ma YX, Wang JQ, Wang J, Wu XH. An interval neutrosophic linguistic multi-criteria group decision–making the method and its application in selecting medical treatment options, *Neural Computer Application.* DOI:10.1007/s00521-016-2203-1. 2016.

[58] A. Chakraborty, S. Broumi, P.K Singh, "Some properties of Pentagonal Neutrosophic Numbers and its Applications in Transportation Problem Environment," *Neutrosophic Sets and Systems*, vol.28, pp.200215, 2019.

[59] A. Chakraborty, S. Mondal, S. Broumi, "De-Neutrosophication technique of pentagonal neutrosophic number and application in minimal spanning tree," *Neutrosophic Sets and Systems*, vol. 29, pp. 1-18, 2019. doi: 10.5281/zenodo.3514383.

Interval Valued Neutrosophic Shortest Path Problem by A* Algorithm

[1] S.Krishna Prabha, [2] Said Broumi,[3] Florentin Smarandache
[1] Assistant Professor, Department of Mathematics,
PSNA College of Engineering and Technology, Dindigul-624622, Tamilnadu, India;,jvprbh1@gmail.com
[2]Laboratory of Information Processing, Faculty of Science Ben M'Sik,
University Hassan II, Casablanca, Morocco ;broumisaid78@gmail.com
[3]Dept. Math and Sciences, University of New Mexico, Gallup, NM, USA; smarand@unm.edu
* Correspondence: jvprbh1@gmail.com

Abstract

Many researchers have been proposing various algorithms to unravel different types of fuzzy shortest path problems. There are many algorithms like Dijkstra's, Bellman-Ford,Floyd-Warshall and kruskal's etc are existing for solving the shortest path problems. In this work a shortest path problem with interval valued neutrosophic numbers is investigated using the proposed algorithm. A* algorithm is extensively applied in pathfinding and graph traversal.Unlike the other algorithms mentioned above, A* algorithm entails heuristic function to uncover the cost of path that traverses through the particular state. In the structured work A* algorithm is applied to unravel the length of the shortest path by utilizing ranking function from the source node to the destination node. A* algorithm is executed by applying best first search with the help of this search, it greedily decides which vertex to investigate subsequently. A* is equally complete and optimal if an acceptable heuristic is concerned. The arc lengths in interval valued neutrosophic numbers are defuzzified using the score function.. A numerical example is used to illustrate the proposed approach.

Keywords: Heuristic function, Interval Valued Neutrosophic Graph, Score Function,Shortest Path Problem. Destination node, Source node.

1.Introduction

In order to overcome the real life situations which could not be handled in some conditions, Zadeh[1] introduced Fuzzy logic which was further developed by Zimmermann[2]. For handling uncertainty the interval valued neutrosophic set is used. The truth-membership, the indeterminacy-membership and the falsity-membership are characterized independently in interval valued neutrosophic set which are able to work with the information's which are conflicting, undetermined, and partial. The rational subdivision of studying the nature, origins, and scope of neutralities, in addition to interface with a variety of ideational spectra is phrased as neutrosophy.The extension of neutrosophic set to neutrosophic offset, underset, and overset was proposed by Smarandache[3]. Bipolar neutrosophic sets, simplified neutrosophic sets, interval valued neutrosophic sets, single valued neutrosophic sets etc are various extension of neutrosophic sets[4,5,6]. The single valued neutrosophic notion is helpful in a range of

DOI: 10.5281/zenodo.4127190

Received: Jun 09, 2020 Accepted: September 03, 2020

fields, such as the decision making problem, medical diagnosis, etc. Various concepts in graph theory were introduced by combining single valued neutrosophic sets. The single valued neutrosophic graph is the simplification of fuzzy graphs and intuitionistic fuzzy graphs. An interval valued neutrosophic graph oversimplifies the notions of a fuzzy graph, an intuitionistic fuzzy graph, an interval valued fuzzy graph and a single valued neutrosophic graph. The most common topic in research is finding the shortest path of the graph by traversing the edges with different types of algorithms. By using the score function Broumi et. al. [7] proposed an algorithm to solve the neutrosophic shortest path problem where the network arc lengths are represented by interval valued neutrosophic numbers. Multiple labeling is applied for finding shortest path with intuitionstic fuzzy arc length by Jabarulla et.al[8]. Kumar and Kaur [9] provided the solution of fuzzy maximal flow problems using fuzzy linear programming. Garg et al. [10] have proposed the Hybrid model for medical diagnosis using Neutrosophic Cognitive Maps with Genetic Algorithms. An Algorithm for shortest path problem in a network with interval valued intuitionstic trapezoidal fuzzy number was presented by Kumar et al.[11]. Jayagowri et al. [12] used Trapezoidal Intuitionistic Fuzzy Number to Find Optimized Path in a Network. Broumi et al have dealt with various concepts of neutroshopic graphs like single valued neutrosophic graphs, on bipolar single valued neutrosophic graphs and interval valued neutrosophic graphs etc with different algorithms [13,14,15,16,17,18]. Various aspects of Neutrosophic Graphs were studied by Smarandache[19]. Pentagonal Neutrosophic Number and its Application in Networking Problem was proposed by Avishek Chakraborty [20]. Thamaraiselvi et al.[21] found a new approach for optimization of real life transportation problems in neutrosophic environment. Tuhin bera[22] gave an approach to solve the linear programming problem using single valued trapezoidal neutrosophic number. Sapan Kumar Das [23] have used neutrosophic numbers in integer programming. Edalatpanah [24] suggested a new technique to solve triangular neutrosophic linear programming . Majumdar et al.[25] has worked on shortest path problem on intuitionistic fuzzy network . Bhimraj basumatary[26] have unraveled the interval-valued triangular neutrosophic linear programming .There are many algorithms existing for solving the shortest path problems like Dijkstra's, Bellman-Ford,Floyd-Warshall and kruskal's etc for finding the optimal path . In this paper A * algorithm is applied for solving the interval valued neutrosophic shortest path problem.

A* algorithm is a best-first search algorithm that depends on an open list and a closed list to discover a path that is both optimal and complete towards the goal. A* search finds the shortest path through a search space to goal state using heuristic function. This technique finds minimal cost solutions and is directed to a goal state called A* search. This algorithm is complete if the branching factor is finite and every action has fixed cost. By defuzzifying the given interval valued neutrosophic cost by applying score function and by applying A* algorithm we find the optimal path.

This paper is organized as follows. In Section 2, the basic concepts about neutrosophic sets and interval valued neutrosophic graph is presented. In Section 3, A* algorithm is proposed to find the shortest path and distance in an interval valued neutrosophic graph. In Section 4 a numerical example is illustrated with the algorithm .Section 5 conclusion and proposals for future research is given.

2. Preliminaries[16]

Definition 2.1:

Let X be a space of points with generic elements in X denoted by x is the neutrosophic set A is an object having the form, A = $\{\langle x : T_A(X), I_A(X), F_A(X)\rangle \,, x \in X\}$,where the functions T,I,F : X→]$^-$0,1$^+$[define respectively the truth-membership function, indeterminacy- membership function and falsity - membership function of the element $, x \in X$ to the set A with the condition $^-0 \leq T_A(X) + I_A(X) + F_A(X) \leq 3^+$ .The functions are real standard or non standard subsets of]$^-$0,1$^+$[.

Definition 2.2:

Let $R_N = \langle [R_T, R_I, R_M, R_E,](T_R, I_R, F_R)\rangle$ and $S_N = \langle [S_T, S_I, S_M, S_E,](T_S, I_S, F_S)\rangle$ be two trapezoidal neutrosophic numbers (TpNNs) and $\theta \geq 0$,then

$R_N \oplus S_N = \langle [R_T + S_T, R_I + S_I, R_M + S_M, R_E + S_E\ ](T_R + +T_s - T_R T_s, I_R I_s, F_R F_S)\rangle$

$R_N \otimes S_N = \langle [R_T . S_T, R_I . S_I, R_M . S_M, R_E . S_E\ ]\big(T_R . T_{s,} I_R + I_s - I_R . I_s, F_R + F_S - F_R . F_S\big)\rangle$

DOI: 10.5281/zenodo.4127190

θ R$_N$ = $\langle[\boldsymbol{\theta} R_T, \boldsymbol{\theta} R_I, \boldsymbol{\theta} R_M, \boldsymbol{\theta} R_E,](1-(1-T_R))^\theta, (I_R)^\theta, (F_R)^\theta)\rangle$

Definition 2.3:

Let X is a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set A (INS A) in X is shown by the truth- membership function T_A (x), an indeterminacy-membership function I_A (x), and a falsity-membership function F_A (x). For each point x in X, there are $T_A(x)$= $[T_A^L, T_A^U] \subseteq [0, 1]$, $I_A(x)$= $[I_A^L, I_A^U] \subseteq [0,1]$, $F_A(x)$= $[F_A^L, F_A^U] \subseteq [0, 1]$, and the sum $T_A(x), I_A(x)$ and $F_A(x)$ satisfies the condition $0 \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3$, then an INS can be expressed as A=$\{\langle x: T_A(x), I_A(x), F_A(x)\rangle, x \in X\}$=$\{\langle x: [T_A^L, T_A^U], [I_A^L, I_A^U], [F_A^L, F_A^U]\rangle, x \in X\}$

Definition 2.4:

Let $\widetilde{\boldsymbol{A}}_1 = \langle[\boldsymbol{T}_1^L, \boldsymbol{T}_1^U], [\boldsymbol{I}_1^L, \boldsymbol{I}_1^U], [\boldsymbol{F}_1^L, \boldsymbol{F}_1^U]\rangle$ and $\widetilde{\boldsymbol{A}}_2 = \langle[\boldsymbol{T}_2^L, \boldsymbol{T}_2^U], [\boldsymbol{I}_2^L, \boldsymbol{I}_2^U], [\boldsymbol{F}_2^L, \boldsymbol{F}_2^U]\rangle$

be two interval valued neutrosophic numbers and l > 0 . Thus, the operational rules are defined as:

1.$\tilde{A}_1 \oplus \tilde{A}_2 = [T_1^L + T_2^L - T_1^L T_2^L, T_1^U + T_2^U - T_1^U T_2^U].[I_1^L I_2^L, I_1^U I_2^U].[F_1^L F_2^L, F_1^U F_2^U]$

2. $\tilde{A}_1 \otimes \tilde{A}_2$=$\langle[T_1^L T_2^L, T_1^U T_2^U].[I_1^L + I_2^L - I_1^L I_2^L, I_1^U + I_2^U - I_1^U I_2^U].[F_1^L + F_2^L - F_1^L F_2^L, F_1^U + F_2^U - F_1^U F_2^U]\rangle$

3. $\lambda \check{A} = \langle[1-(1-T_I^L)^\lambda, 1-(1-T_I^U)^\lambda].[(I_1^L)^\lambda.(I_1^U)^\lambda].[(F_1^L)^\lambda.(F_1^U)^\lambda]\rangle$

4. $\tilde{A}^\lambda$=$\langle[(T_1^L)^\lambda, (T_1^U)^\lambda].[1-(1-I_I^U)^\lambda.1-(1-I_I^U)^\lambda],[1-(1-F_I^L)^\lambda, 1-(1-F_I^U)^\lambda]\rangle$,Where $\lambda > 0$

Definition 2.5: To compare between two IVNN, Ridvan [33] used a score function concept in 2014. The score function is used for comparing the IVNS grades. This function demonstrates that the greater the value, the greater the interval-valued neutrosophic sets, and through the use of this concept paths can be ranked.

Let $\tilde{A}_1 = (T_1, I_1, F_1)$ be an interval valued neutrosophic number, then, the score function $s(\widetilde{\boldsymbol{A}}_1)$ of an IVNN can be defined as follows:

$$S(\hat{A}_1) = \left(\frac{1}{4}\right) \times [2 + T_1^L + T_1^U - 2I_1^L - 2I_1^U - F_1^L - F_1^U]$$

Comparison of interval valued neutrosophic numbers

Let $\tilde{A}_1 = (T_1, I_1, F_1)$ and $\tilde{A}_2 = (T_2, I_2, F_2)$ be two interval valued neutrosophic numbers then

(i) $\hat{A}_1 < \hat{A}_2$ if $S(\hat{A}_1) < S(\hat{A}_2)$

(ii) $\hat{A}_1 > \hat{A}_2$ if $S(\hat{A}_1) > S(\hat{A}_2)$

(iii) $\hat{A}_1 = \hat{A}_2$ if $S(\hat{A}_1) = S(\hat{A}_2)$

3. Basic notations in A* search

f – f is the parameter of A* which is the sum of the other parameters G and H and is the least cost from one node to the next node. This parameter is responsible for helping us find the most optimal path from our source to destination.

g – g is the cost of moving from one node to the other node. This parameter changes for every node as we move up to find the most optimal path.

h – h is the heuristic/estimated path between the current code to the destination node. This cost is not actual but is, in reality, a guess cost that we use to find which could be the most optimal path between our source and destination

3.1: Algorithm for A* Search

1. Locate the initial node on the list of ORIGIN.
2. If (ORIGIN is empty) or (ORIGIN = GOAL) terminate search.
3. Remove the first node from ORIGIN. Call this node as n.
4. If (n = GOAL) terminate search with success.
5. Else in case, if node a has successors, generate all of them. Find the fitness number of the successors by totaling the evaluation function value & the cost function value. Sort the list by fitness number.
6. Name the list as START.
7. Replace ORIGIN with START.

DOI: 10.5281/zenodo.4127190

8. Go to step 2.

3.2: A* Pseudo Code

Let us have the following assumptions,

Let us denote the goal node as ng , node_current as nc , node_successor as nsc ,successor_current_cost as scc and source node is ns.

The nodes that have been evaluated by the heuristic function but not expanded into successors yet are collected in OPEN set.

The nodes that have been visited and expanded in CLOSE set.

1. Put ns in the OPEN list with f(ns) = h(ns) (initialization)
2. While the OPEN list is not empty {
3. Take from the open list the node nc with the lowest
4. f(nc) = g(nc) ⊕ h(nc)
5. if nc is ng we have found the solution; break
6. Generate each state nsc that come after nc
7. for each nsc of nc {
8. Set scc = g(nc) ⊕ w(nc, nsc)
9. if ns is in the OPEN list {
10. if g(ns) ≤ scc then go to (line 20)
11.} else if nsc is in the CLOSED list {
12. if g(nsc) ≤ scc then go to (line 20)
13. Move nsc from the CLOSED list to the OPEN list
14.} else {
15. Add nsc to the OPEN list
16. Set h(nsc) to be the heuristic distance to ng
17.}
18. Set g(nsc) = scc
19. Set the parent of nsc to nc
20.}
21. Add nc to the CLOSED list.
22.}
23. if(nc != ng) exit with error (the OPEN list is empty)

4. **Numerical Example**

Consider the given interval valued neutroshopic shortest path problem with five edges with interval valued neutroshopic fuzzy weights as in Fig:1.

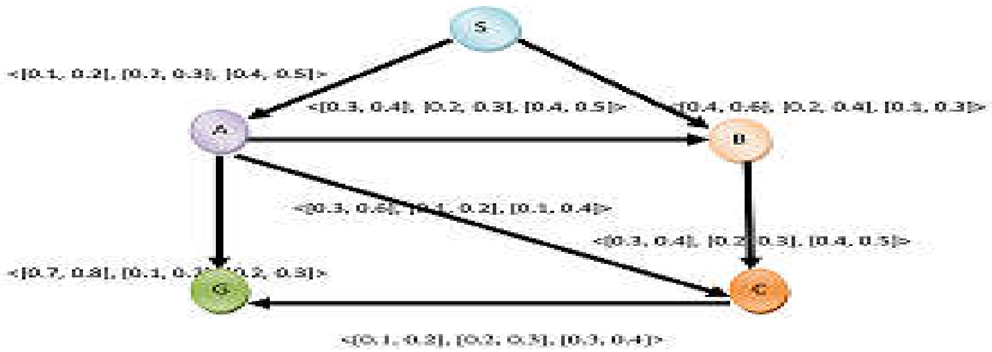


Fig.1.An interval valued neutroshopic shortest path problem

Let us take the interval valued neutroshopic fuzzy weight for the edge from S-A, by applying the score function formula we convert the interval valued neutroshopic fuzzy weights to crisp number,

<[0.1, 0.2], [0.2, 0.3], [0.4, 0.5]>

$S((A) = \left(\frac{1}{4}\right) \times [2 + T_1^L + T_1^U - 2I_1^L - 2I_1^U - F_1^L - F_1^U]$

$= \left(\frac{1}{4}\right) \times [2 + 0.1 + 0.2 - 2 \times 0.2 - 2 \times 0.3 - 0.4 - 0.5]$

$= \left(\frac{1}{4}\right) \times [2.3 - 0.4 - 0.6 - 0.4 - 0.5]$

$= \left(\frac{1}{4}\right) \times [0.4] = 0.1$

Similarly by proceeding with the formula for score function, we can find the crisp values given in the table below.

Table:1. Interval valued Neutrosophic distance

Edges	Interval valued Neutrosophic distance	Crisp Values
S-A	<[0.1, 0.2], [0.2, 0.3], [0.4, 0.5]>	0.1
S-B	<[0.4, 0.6], [0.2, 0.4], [0.1, 0.3]>	0.35
A-B	<[0.3, 0.4], [0.2, 0.3], [0.4, 0.5>	0.2
B-C	<[0.3, 0.4], [0.2, 0.3], [0.4, 0.5>	0.2
A-C	<[0.3, 0.6], [0.1, 0.2], [0.1, 0.4]>	0.45
A-G	<[0.7, 0.8], [0.1, 0.2], [0.2, 0.3]>	0.6
C-G	<[0.1, 0.2], [0.2, 0.3], [0.3, 0.4]>	0.15

After finding the crisp values and substituting in the corresponding paths we get,

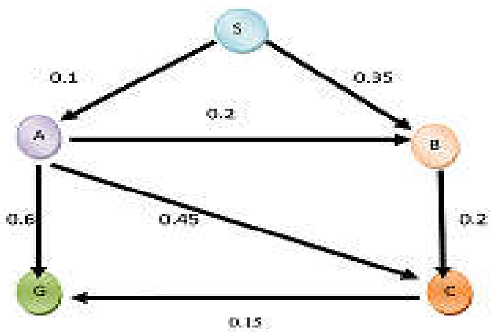


Fig.2. Crisp valued neutroshopic shortest path problem

Interval valued neutroshopic heuristic values to end nodes are given in the following table.2.

DOI: 10.5281/zenodo.4127190

Table: 2. Heuristic values

Node	h(n)	Crisp h(n)
S	<[0.9, 0.8], [0.1, 0.2], [0.2, 0.3]>	0.65
A	<[0.7, 0.8], [0.1, 0.2], [0.2, 0.3]>	0.6
B	<[0.3, 0.4], [0.2, 0.3], [0.4, 0.5]>	0.2
C	<[0.1, 0.2], [0.2, 0.3], [0.4, 0.5]>	0.1
G	<[0,0], [0,0], [1, 1]>	0

Let us start with the source node S

ITERATION: 0

S→0⊕0.65=0.65

From the source node S the graph expands through two paths A and B.

ITERATION: 1

S→A: f (A) = g(A) ⊕h(A) = 0.1⊕0.6=0.7

S→B: f (A) = g (B) ⊕h(B) = 0.35⊕0.2=0.55(MIN)

By comparing the above values the path S→B has minimum value, so we proceed to traverse from that path,

ITERATION: 3

S→B →C: f(A) =g(C) ⊕h(C) = (0.35⊕0.2)⊕0.1=0.65

S→A →B: f(B) =g (B) ⊕h (B) = (0.1⊕0.2) ⊕0.2=0.5(MIN)

S→A →C: f(C) = g(C) ⊕h(C)=(0.1⊕0.45)⊕0.1=0.65

S→A →G: f(G) = g(G) ⊕h(G) = (0.1⊕0.6)⊕0=0.7

By comparing the above values the path S→A →B has minimum value, so we proceed to traverse from that path,

ITERATION: 4

S→A →B→C: f(C) = g(C) ⊕h(C) = (0. 1⊕0.2⊕0.2) ⊕0.1=0.6(MIN)

By comparing the paths S→A →B→C and S→B the path S→B is minimum.

So traverse from that path to reach the goal node.

ITERATION: 5

S→B →C→G: f (G) = g(G) ⊕h(G)=(0.35⊕0.2⊕0.15)⊕0=0.7

By comparing the paths S→B →C→G and S→A →B→C the path S→A →B→C is minimum.

So traverse from that path to reach the goal node.

ITERATION: 6

To reach the goal node we can traverse in the two paths,

S→A →B→C→G: f(G)= g(G) ⊕h(G)=(0.1⊕0.2⊕0.2⊕0.15)⊕0=0.65(MIN)

S→A →C→G: f(G)= g(G) ⊕h(G)=(0.1⊕0.45⊕0.15)⊕0=0.7

By comparing the paths S→A →B→C→G and S→A →C→G the path S→B →C→G is minimum

DOI: 10.5281/zenodo.4127190

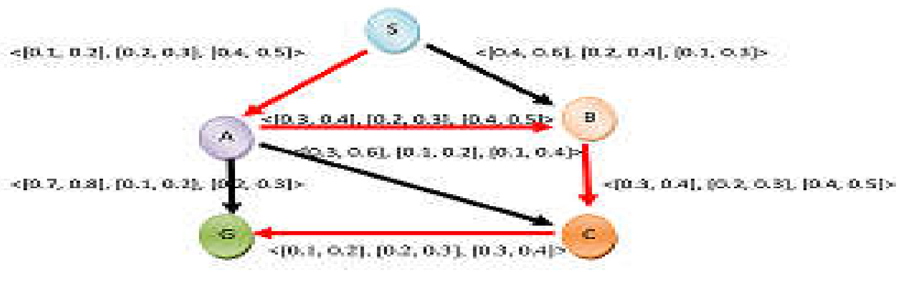


Fig.3. Shortest path from source node to goal node.

The shortest path is given by **S→A →B→C→G.**

5. Conclusion

A* algorithm is applied to solve the shortest path problem on a network with an interval valued neutrosophic arc lengths in this paper. A* algorithm is complete and optimal. A* algorithm is the best one from other techniques. It is used to solve very complex problems. A* algorithm is optimally efficient, i.e. there is no other optimal algorithm guaranteed to expand fewer nodes than A*.Heuristic values are considered in calculating the path by this method. Score function is used for defuzzification. The technique is enlightened by a numerical example with the help of theoretical information. The time complexity of A* depends on the heuristic. In the worst case of an unbounded search space, the number of nodes expanded is exponential in the depth of the solution (the shortest path) d: $O(b^d)$, where b is the branching factor. A* can also be adapted to a bidirectional search algorithm.Furthermore, the following algorithm of the interval neutrosophic shortest path problem can be extended into an interval valued bipolar neutrosophic environment.

Funding: “This research received no external funding”

Conflicts of Interest: “The authors declare no conflict of interest.”

References

1. L.Zadeh, “Fuzzy sets”, Inform and Control, 8, 338-353, 1965.
2. H. J. Zimmermann, “Fuzzy set Theory - and Its Applications”, Kluwer-Nijhoff Publishing, Boston-Dordrecht-Lancaster, 1985.
3. Florentin Smarandache, “Neutrosophic Overset, Neutrosophic Underset, and Neutrosophic Offset”. Similarly for Neutrosophic Over-/Under-/Off- Logic, Probability, and Statistics, 168 p., Pons Editions, Bruxelles, Belgique.2016.
4. S.Broumi, A.Bakali, M.Talea, F.Smarandache, “On Bipolar Single Valued Neutrosophic Graphs”, Journal of New Theory, N11, pp.84-102, 2016.
5. S.Broumi, A,Bakali, M.Talea, F.Smarandache , “ Single Valued Neutrosophic Graphs”, Journal of New

Theory, N 10, pp. 86-101,2016.

6. S.Broumi, A. Bakali, M.Talea, F.Smarandache, "Single Valued Neutrosophic Graphs: Degree, Order and Size", IEEE International Conference on Fuzzy Systems (FUZZ), pp.2444-2451,2016.
7. S.Broumi, A,Bakali, M.Talea, F.Smarandache , L.Vladareanu, "Computation of Shortest Path Problem in a Network with SV-Trapezoidal Neutrosophic Numbers", Proceedings of the 2016 International Conference on Advanced Mechatronic Systems, Melbourne, Australia, pp.417-422,2016.
8. A.Ngoor and M.Jabarulla , "Multiple labeling Approach For Finding shortest Path with Intuitionstic Fuzzy Arc Length", International Journal of Scientific and Engineering Research,V3,Issue 11,pp.102-106,2012.
9. A.Kumar and M.Kaur, "Solution of fuzzy maximal flow problems using fuzzy linear programming",World Academy of Science and Technology.87,28-31,2011.
10. Gaurav, & Kumar, Megha & Bhutani, Kanika & Aggarwal, Swati, "Hybrid model for medical diagnosis using Neutrosophic Cognitive Maps with Genetic Algorithms". 1-7. 10.1109/FUZZ-IEEE.2015.7338015,2015.
11. G.Kumar, R.K.Bajaj and N.Gandotra, "Algoritm for shortest path problem in a network with interval valued intuitionstic trapezoidal fuzzy number", Procedia Computer Science, 70, pp.123-129, 2015.
12. P.Jayagowri and G.Geetha Ramani, "Using Trapezoidal Intuitionistic Fuzzy Number to Find Optimized Path in a Network", Advances in Fuzzy Systems., Article ID 183607, 6 pages http://dx.doi.org/10.1155/2014/183607,2014.
13. S.Broumi, A,Bakali, M.Talea, F.Smarandache and L.Vladareanu, "Applying Dijkstra for Solving Neutrosophic Shortest Path Problem", Proceedings of the 2016 International Conference on Advanced Mechatronic Systems, Melbourne, Australia, pp.412-416,2016.
14. S.Broumi, F.Smarandache, "New distance and similarity measures of interval neutrosophic sets, Information Fusion (FUSION)", IEEE 17th International Conference, pp 1 –7,2014.
15. S.Broumi, M.Talea, A.Bakali, F.Smarandache, "An Introduction to Bipolar Single Valued Neutrosophic Graph Theory". Applied Mechanics and Materials, vol.841, 184-191.2016.
16. S.Broumi, M.Talea, A.Bakali, F.Smarandache , K. P.Kishore,Şahin, " Shortest Path Problem under Interval Valued Neutrosophic Setting", International Journal of Advanced Trends in Computer Science and Engineering, volume 8,pg216-222,2019.
17. S.Broumi, A.Bakali, M.Talea, F.Smarandache, "Isolated Single Valued Neutrosophic Graphs". Neutrosophic Sets and Systems, Vol. 11, pp.74-78, 2016.
18. S.Broumi, A.Bakali, M.Talea, F.Smarandache, "Interval Valued Neutrosophic Graphs", Critical Review, XII, pp.5-33, 2016.
19. F.Smarandache,"Types of Neutrosophic Graphs and neutrosophic AlgebraicStructures together with their Applications in Technology", seminar, Universitatea Transilvania din Brasov, Facultatea de Design de Produs si Mediu, Brasov, Romania .2015.
20. Avishek Chakraborty, "A New Score Function of Pentagonal Neutrosophic Number and its Application in Networking Problem", International Journal of Neutrosophic Science, Volume 1, Issue 1, PP: 40-51, 2020.

21. R.Thamaraiselvi and R. Santhi, “A New Approach for Optimization of Real Life Transportation Problems in Neutrosophic Environment”, Mathematical Problems in Enginering.Article ID 5950747,9http://dx.doi.org/10.1155/2016/5950747,2016.
22. Tuhin Bera, Nirmal Kumar Mahapatra, “An Approach to Solve the Linear Programming Problem Using Single Valued Trapezoidal Neutrosophic Number”, International Journal of Neutrosophic Science ,Vol. 3, No. 2, PP. 54-66, 2020.
23. Sapan Kumar Das, S.A. Edalatpanah, “ A new ranking function of triangular neutrosophic number and its application in integer programming”, International Journal of Neutrosophic Science,Volume 4 , Issue 2, PP: 82-92 , 2020.
24. S.A.Edalatpanah, “A Direct Model for Triangular Neutrosophic Linear Programming”, International Journal of Neutrosophic Science Volume 1 , Issue 1, PP: 19-28 , 2020.
25. S.Majumdar and A.Pal, “Shortest Path Problem on Intuitionistic Fuzzy Network”, Annals of Pure and Applied Mathematics, Vol. 5, 1, 26-36, 2013.
26. Bhimraj Basumatary, Said Broumi, “Interval-Valued Triangular Neutrosophic Linear Programming Problem”, International Journal of Neutrosophic Science, Volume 10, Issue 2, PP: 105-115, 2020.

DOI: 10.5281/zenodo.4127190

An Analysis on Novel Corona Virus by a Plithogenic Approach to Fuzzy Cognitive Map

R.Sujatha[1,*], S.Poomagal[2], G.Kuppuswami[3] and Said Broumi [4]
[1]Department of Mathematics, Sri Sivasubramaniya Nadar College of Engineering, Chennai, INDIA;
sujathar@ssn.edu.in
[2]Department of Mathematics, University College of Engineering – Kancheepuram, Kancheepuram,INDIA;
poomagal777@gmail.com
[3]Department of Mathematics, Sri Venkateswaraa College of Technology, Sriperumbudur, INDIA.
gkuppuswamiji@gmail.com
[4]Laboratory of Information Processing, Faculty of Science Ben MSik, University Hassan II, Casablanca, Morocco.
e-mail:broumisaid78@gmail.com

* Correspondence: sujathar@ssn.edu.in

Abstract

In this paper a Plithogenic approach to Fuzzy cognitive map has been proposed to analyse the impact of novel corona virus.Contradiction degree is an advantageous feature of Plithogenic sets which highly deals with the uncertainty and it substantially increases the accuracy of results. In this research, a new approach is proposed to accumulate the opinions of experts.Using contradiction degree between opinions of experts and plithogenicoperator,the connection matrices obtained from distinct experts are aggregated to improvise a degree of uncertainty. An Analysis on covid-19 (causes, spread and precaution) is done based on the proposed methodology.

Keywords: Fuzzy cognitive maps,Plithogenic sets, Covid-19, Plithogenic operators

1.Introduction

The outbreak of the 2019 coronavirus (COVID-19) infections has spread worldwide, causing fever, severe respiratory illness and pneumonia. The virus is related to severe acute respiratory syndrome coronavirus (SARS-CoV) [27], as compared to MERS-CoV, and SARS-CoV, COVID-19 exhibits faster human transmission, leading to the declaration of a world-wide health emergency by world health organisation (WHO) [28]. No specific drugs or vaccines are available till date. The Clinical evidences of COVID-19 are characterized by cough, fever, bilateral infiltrates on chest imaging. After testing positive, majority of the affected individuals were experiencing moderate symptoms, whereas 20% of the affected people show severe respiratory failure and septic shock [29], gastrointestinal, lymphopenia, myalgias, and lung abnormalities [30]. The death causing ability of the virus is dependent on chronic disorders [29]. The infection has been reported to cause moralities in aged persons [32].

Axelrod [1] has initiated the concept of cognitive maps (CM), a directed graph which represents the causal relationship between the data in a specific field. In the cognitive maps the concepts and causal relationship are represented by the nodes and edges respectively. The edges have the weights representing the intensity of the causal relationship among the concepts. Regardless, CMs applicability is restrained as it has the limitation of disability to

DOI: 10.5281/zenodo.4127202

Received: Jun 13, 2020 Accepted: September 25, 2020

define the strength of interrelationship between the concepts/factors. To overcome the inadequacy, kosko[2] with the zadeh's concept of fuzzy[3] introduced fuzzy cognitive maps, as an extension of conventional CMs by defining the strength by fuzzy numbers. To construct the FCM the experts opinions were obtained based their experience in their field and the strength of the causal relationship between the factors can be estimated [4]. Several approaches are available for the specification of weights in FCM, in order to overcome the difficulty of assigning a crisp real number to express their views with regard to the strength of relationships which are uncertain, linguistic variables were preferred [5]. Triangular Fuzzy Numbers (TFNs) were used to represent relationships between the concepts instead of fuzzy singletons to get by efficiently with uncertainty and practice of fuzzy numbers will lead to more precise and information wealthy FCMs than the conventional FCMs[6] [7][8].FCM modelling has a wide range of application in the field of medicine and extensively reviewed in [20]. FCM was developed for medical decision support systems[21][22], prediction of probability occurance of stroke[23],for risk management in breast cancer[24], to define challenges and articulating solutions in nursing discipline[25] and to study the symptoms of migraine[26].

Fuzzy cognitive maps is extended to plithogenic fuzzy cognitive map (PFCM) which integrates the contradiction degree to the concepts and the methodology of FCM[9]. Smarandache [10] introduces the concept of Plithogenic sets, a generalization of crisp, fuzzy, intuitionistic fuzzy and neutrosophic sets. Plithogenic sets rely on the principle of quantification of the qualitative aspects and degree of contradiction is also the distinguishing feature of Plithogenic sets.Abdel-Basset integrates the concept of Plithogenic sets in decision making for selecting supply chain sustainability metrics[11],evaluation of hospital medical care systems[12] and for evaluating the performance of IoT based supply chain[13], Plithogenic MCDM based onTOPSIS –CRITIC model for sustainable supply chain risk management[14].In another study, Plithogenic sets used for multi variable data analysis in [32-35].FCM depends totally on the expert's opinion, itlapse due to its uncertainty associate with the responses of each experts and contradiction between them.

This paper proposes a Plithogenic approach to fuzzy cognitive map to reform the drawback. The proposed model follows the methodology of FCM, in addition it merges the experts' opinionby using the Plithogenic operators, based on the contradiction degree between them.

2. Materials and methods

2.1 Fuzzy cognitive maps

Fuzzy cognitive maps(FCMS) are well developed computational method which combines the neural networks and fuzzy logic. FCMs are the fuzzy directed graphs with nodes representing the factors of the system considered and edges represents the causal relationship between the concepts/factors. The edges are characterised by the weights w_{ij}, which represents the strength of the causality between the concepts.The values of FCM are fuzzy, so the weights are represented in terms of linguistic variables [15]. Fuzzy linguistic scales are then interpreted to the fuzzy numbers through the information provided by the experts. A simple example of FCM is depicted in Figure-1.

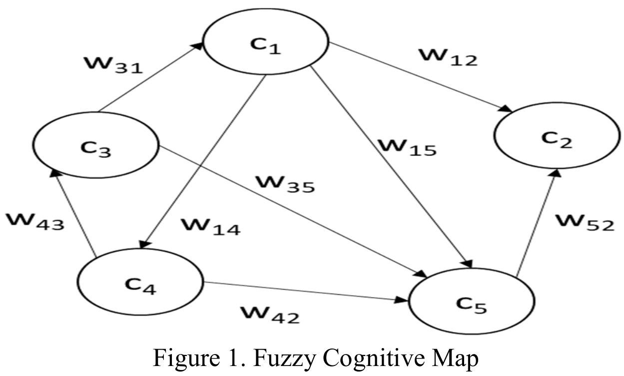


Figure 1. Fuzzy Cognitive Map

DOI: 10.5281/zenodo.4127202

The influence of an individual concept on other concepts of the system is calculated by using the equation (1), the inference of FCM.

$$A(t+1) = f(A(t) * TrF(M)) \tag{1}$$

where $A(t+1)$ and$A(t)$ is the strength of the concept C_i at step t+1 and t respectively and$TrF(M)$ is a connection matrix containing the strengths of the causalty between the concepts. Here$f$ represents the threshold function,for activating the successive vector after each pass and to settle down to a fixed point which is the hidden pattern of the system for that corresponding state vector [16]. The iteration proceeds until the limit cycle [16] obtained.Though there were several threshold functions available,a conventional threshold function is considered. To establish the causal influence among the concepts fuzzy inference with IF THEN ruleused is as follows:

IF C_i is ON THEN C_j is A(The influence of C_i on C_j is A) (2)

2.2 Triangular Fuzzy number

A fuzzy number generalises the concept of crisp numbers. For simplicity and accuracythe scales of the linguistic variables are interrelated with the Triangular fuzzy numbers, triangular form can easily grasp the uncertainty in the human perception with better approximation [18].

The fuzzy number $\tilde{A} = (a,b,c)$is a triangular fuzzy number if its membership function is $\mu_{\tilde{A}}(x) =$

$$\begin{cases} 0 & x < a \\ \dfrac{x-a}{b-a} & a \le x \le b \\ \dfrac{c-x}{c-b} & b \le x \le c \\ 0 & x \ge c \end{cases} \tag{3}$$

The triangular fuzzy number is graphically represented in Figure 2.

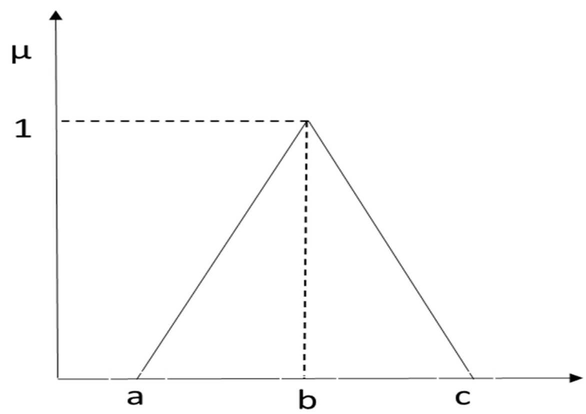


Figure-2 Triangular fuzzy number with parameters (a,b,c)

2.3 Plithogenic operator

Plithogenic set a generalisation of the crisp, fuzzy, intuitionistic fuzzy and neutrosophicsets,is highly focused on uncertainty of the information. The two main features of plithogenic sets are its contradiction degree and degree of appurtenance. The contradiction degree is defined between the attribute values to the dominant attribute value and in fact it is the core elementof plithogenic aggregation operators which increases the accuracy of results. This advantage is the motivation for current study which integrates the plithogenic aggregation operator to combine the connection matrix obtained from the experts' opinion.Plithogenic operators are defined as

DOI: 10.5281/zenodo.4127202

Plithogenic fuzzy set intersection:

$$a \wedge_P b = (1-c)[a \wedge_F b] + c[a \vee_F b] \quad (4)$$

Plithogenic fuzzy set union:

$$a \vee_P b = (1-c)[a \vee_F b] + c[a \wedge_F b] \quad (5)$$

where a and b are the degree of appurtenance of the attribute with respect to the expert A and expert B respectively, c represents the contradiction degree, $a \wedge_F b$ and $a \vee_F b$ are the fuzzy t-norms and t-co-norms respectively.

2.4 Defuzzification method

Defuzzification is the process of converting the fuzzified value to the crisp value and is done by a centroid based method[19].

3. **Proposed methodology**

In this paper, a plithogenic approach to a fuzzy cognitive mapis proposed for an analysis on covid-19 which deals with the imprecise data. In this work, the opinion obtained from three different experts has been combined by plithogenic operators and the analysis is done by fuzzy cognitive maps modelling. Plithogenic set plays an evident role in handling imprecise judgements by taking both truth and false degrees of membership [13]. In addition plithogenic operator has the advantage of contradiction degree which ensures the more accurate results thanpreviously existing models. As far as medical field is concern it is very necessary to get opinion from more than one expert to reach the conclusion for any kind of disease. Different experts have different opinions regarding the Covid-19 spreading and controlling, Plithogenic set combines all the expertsopinion to analyze the virus Covid-19 via possibility of spreading and controlling measures. This is entirely a different way of analysing Covid-19 mathematically using Plithogenic sets. In this aspect,the current study interprets the benefits of Plithogenicoperator and fuzzy cognitive map modelling.Since the fuzzy cognitive map's principle totally rely on the expert's opinion, the authors have considered the contradiction degree for the experts instead of contradiction degree between the factors, to improve the results and also to accommodate the efficiency of Plithogenic operator.

3.1 Determining the factors and construction of FCM for analysis

Based on the field of system considered, select a panel of expert members to monitor the evaluation process. Obtain the concepts/factors of the system from the expert members and construct the FCM by defining the nodes, edges and their weights representing the strength of the interrelationship between the concepts. The experts prefer to afford their opinion in terms of linguistic variables rather than the crisp values due to uncertainties associated with factors of the system considered. Linguistic variables are interpreted to the fuzzy linguistic values from the linguistic scales obtained from the expert members. In this study the linguistic values assigns triangular fuzzy numbers for its accuracy and simplicity. From the data acquired, connection matrix is constructed for every individual expert's opinion whose entries are in terms of linguistic variables.$CM(E_k) = (x_{ij})_{n\times n}$a connection matrix, where x_{ij} is the linguistic expression obtained from the expert(E_k)opinion representing influence of concept C_i on C_j.

3.2 Plithogenic aggregation of weights

In the aspect of increasing an accuracy of results, plithogenic operators are utilized to accumulate the opinions of experts. This study determines the contradiction degree between the experts instead of attributes [9][12]. Let us compute the aggregation matrix of the system by using plithogenic operators (3) rely on contradiction degree between experts opinion. $M = (y_{ij})$, where y_{ij} is strength of the influence between concept C_i on C_j obtained by accumulating the values obtained from all the experts.

DOI: 10.5281/zenodo.4127202

3.3 Determining the influence of concepts

Fuzzy cognitive map approach is used for simulation of results. To determine the influence of an individual concept on every other concepts of the system, initiate an instantaneous state vector by keeping the corresponding concept in ON position. Passing the instantaneous vector on to the connection matrix, a resultant vector is obtained and is updated by applying the threshold function. This paper adapts conventional threshold function. The updated vector is again passed on to connection matrix and the procedure is repeated until the fixed point attains, a limiting cycle of the FCM [16].

3.4 Algorithm of the proposed model

The steps involving in the proposed model are as follows and is pictorially presented in Figure3.

Step1: Consider the evaluation system.

Step2: Form a committee of expert members in the field of a system considered.

Step3: From the experts' knowledge, obtain the concepts for the construction of FCM and linguistic expressions representing strength of the interrelationship between the concepts.

Step 4: Form the connection matrices from a data acquired from the experts whose entries are linguistic variables. The linguistic variables are then interpreted to a triangular membership values, formed from the linguistic scales given by the experts.

Step 5: The connection matrices are then accumulated to a single matrix using plithogenic operator equation (4).

Step 6: To determine the effect a concept (say(N_1)) on other factors of the system, consider the initial state vector by kept N_1 in ON and remaining concepts in OFF state.

Step 7: The influence of the concepts of the system is calculated by using the equation $A(t+1) = f(A(t) * TrF(M))$, the inference of FCM. Threshold the resultant vector to activate the successive vectors. The vector updated is passed on to the connection matrix and the process is repeated till fixed point is arrived.

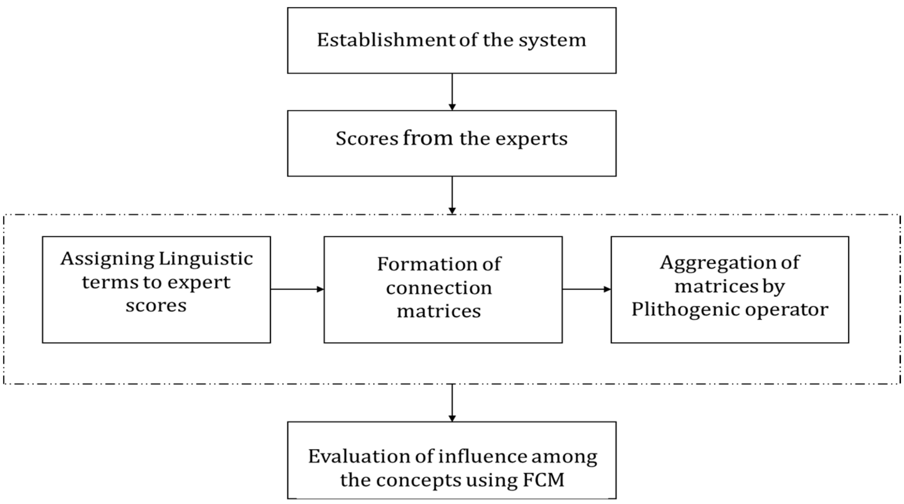


Figure-3 Flow Chart of Proposed Methodology

DOI: 10.5281/zenodo.4127202

4.Application of Plithogenic FCM in Analysis of Covid-19 (causes, spread and precautions)

The outbreak of covid-19 is considered for the analysis, to test the accuracy of the proposed methodology.This work mainlyfocuses on causes, spread and precaution.

4.1 Factors considered for analysis

The factors/concepts considered for the analysis was obtained from the experts and is listed below.

- N1-Symptomatic Covid-19(Symptoms like cold, cough, fever and breathing problems).
- N2-Asymptomatic Covid-19(No symptoms mentioned in N1).
- N3-Maintaining social distance, wearing mask and continuous hand wash.(Precaution measure to prevent from Covid-19).
- N4-Chronic disease patients (High blood pressure, diabetes, tuberculosis, cancer patient, elder people) violating N_3.
- N5- Travelling history(Person travelling from country to country).
- N6-Possibility of Covid-19.
- N7-High risk factor for getting Covid-19
- N8-Prevention measures from Covid-19

4.2 Results and Discussion

For this study three experts were asked to provide their opinion on covid-19 for the analysis, the connection matrix is constructed based on the data obtained from the knowledge of experts and their field experiences. The cognitive maps drawn from the opinions ofexpert1,expert2 and expert3 are depicted in Figure-3, Figure-4 and Figure-5 respectively, theirconnection matrices are presented in equation (5),(6) and(7) respectively. The entries are presented in terms of linguistic variables. Using triangular fuzzy numbers the linguistic variables are quantified and presented in Table1 and is graphically presented in Figure-4.

Linguistic variable	Linguistic value
Very Low	(0,0.2,0.3)
Low	(0.2,0.3,0.4)
Medium	(0.3,0.4,0.5)
High	(0.5,0.6,0.7)
Very High	(0.7,0.8, 1)

Table-1 Quantified linguistic variables

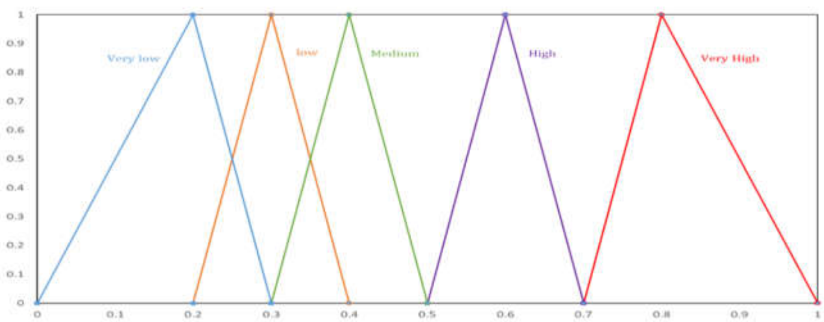


Figure-4 Triangular membership values

DOI: 10.5281/zenodo.4127202

$$CM(E_1) = \begin{array}{c} \\ N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \\ N_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} N_1 & N_2 & N_3 & N_4 & N_5 & N_6 & N_7 & N_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & H & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & H & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & VL & L & H \\ 0 & 0 & 0 & 0 & 0 & H & H & 0 \\ 0 & 0 & 0 & 0 & 0 & H & VH & 0 \\ H & H & VL & H & H & 0 & 0 & 0 \\ 0 & 0 & L & H & VH & 0 & 0 & 0 \\ 0 & 0 & H & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(6)

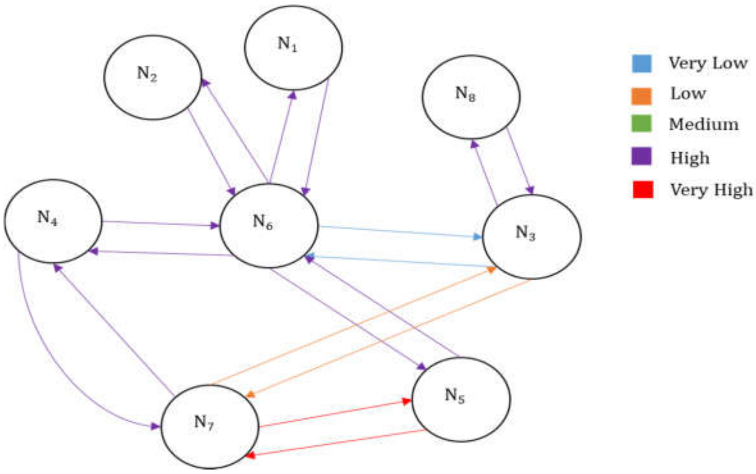


Figure-5 Fuzzy Cognitive Map(Expert 1)

$$CM(E_2) = \begin{array}{c} \\ N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \\ N_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} N_1 & N_2 & N_3 & N_4 & N_5 & N_6 & N_7 & N_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & VH & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & M & L & 0 \\ 0 & 0 & 0 & 0 & 0 & VL & VL & VH \\ 0 & 0 & 0 & 0 & 0 & VH & VH & 0 \\ 0 & 0 & 0 & 0 & 0 & VH & H & 0 \\ VH & M & VL & VH & VH & 0 & 0 & 0 \\ 0 & L & VL & VH & H & 0 & 0 & 0 \\ 0 & 0 & VH & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \quad (7)$$

DOI: 10.5281/zenodo.4127202

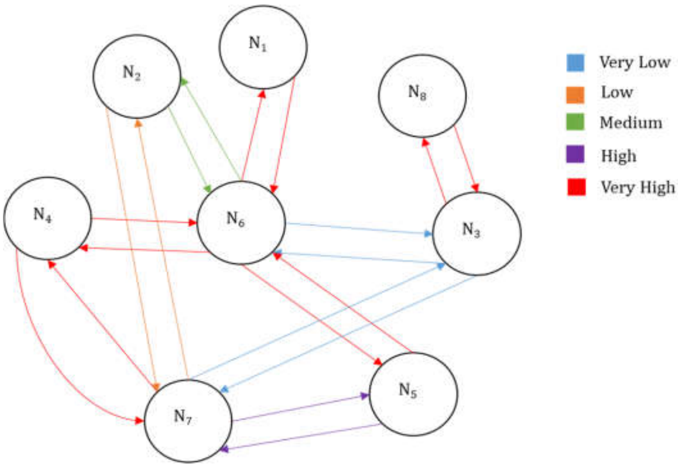


Figure-6 Fuzzy Cognitive Map(Expert 2)

$$CM(E_3) = \begin{array}{c} \\ N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \\ N_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} N_1 & N_2 & N_3 & N_4 & N_5 & N_6 & N_7 & N_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & VH & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & VH & VL & 0 \\ 0 & 0 & 0 & 0 & 0 & VL & L & H \\ 0 & 0 & 0 & 0 & 0 & H & VH & 0 \\ 0 & 0 & 0 & 0 & 0 & VH & VH & 0 \\ H & VH & VL & H & H & 0 & 0 & 0 \\ 0 & VL & L & VH & VH & 0 & 0 & 0 \\ 0 & 0 & VH & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \tag{8}$$

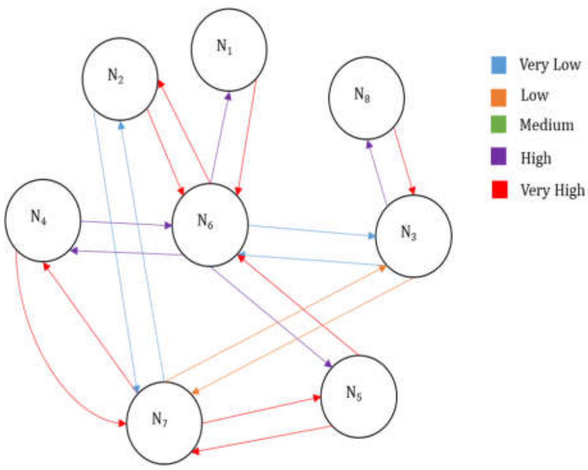


Figure-7 Fuzzy Cognitive Map(Expert 3)

Meanwhile Expert1's opinion is more appropriate than other two experts. Thus the contradiction degree of the dominant expert with respect to the other experts are considered as $d(E_1) = 0, d(E_2) = \frac{1}{3}$ and $d(E_3) = \frac{2}{3}$. The three matrices obtained from experts are accumulated using Plithogenic operator defined in equation (5). The aggregated matrix and the corresponding defuzzified matrix is given in Eqn (9) and Eqn (10) respectively.

DOI: 10.5281/zenodo.4127202

$$
\begin{array}{c}
\\ N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \\ N_8
\end{array}
\begin{array}{c}
\begin{array}{cccccccc} N_1 & N_2 & N_3 & N_4 & N_5 & N_6 & N_7 & N_8 \end{array} \\
\left[\begin{array}{cccccccc}
0 & 0 & 0 & 0 & 0 & (0.656\;\;0.757\;\;0.933) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & (0.585\;\;0.689\;\;0.856) & (0.04\;\;0.167\;\;0.24) & 0 \\
0 & 0 & 0 & 0 & 0 & (0\;\;0.2\;\;0.3) & (0.153\;\;0.278\;\;0.378) & (0.656\;\;0.757\;\;0.933) \\
0 & 0 & 0 & 0 & 0 & (0.545\;\;0.647\;\;0.767) & (0.656\;\;0.757\;\;0.933) & 0 \\
0 & 0 & 0 & 0 & 0 & (0.656\;\;0.757\;\;0.933) & (0.656\;\;0.757\;\;0.933) & 0 \\
(0.545\;\;0.647\;\;0.767) & (0.585\;\;0.689\;\;0.856) & (0\;\;0.2\;\;0.3) & (0.545\;\;0.647\;\;0.767) & (0.545\;\;0.647\;\;0.767)0.653 & 0 & 0 & 0 \\
0 & (0.04\;\;0.167\;\;0.24) & (0.153\;\;0.278\;\;0.378) & (0.656\;\;0.757\;\;0.933) & (0.656\;\;0.757\;\;0.933) & 0 & 0 & 0 \\
0 & 0 & (0.656\;\;0.757\;\;0.933) & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\end{array}
\qquad (9)
$$

$$
TrF(M) =
\begin{array}{c}
\\ N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \\ N_8
\end{array}
\begin{array}{c}
\begin{array}{cccccccc} N_1 & N_2 & N_3 & N_4 & N_5 & N_6 & N_7 & N_8 \end{array} \\
\left[\begin{array}{cccccccc}
0 & 0 & 0 & 0 & 0 & 0.782 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.711 & 0.149 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.17 & 0.27 & 0.782 \\
0 & 0 & 0 & 0 & 0 & 0.653 & 0.782 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.782 & 0.782 & 0 \\
0.653 & 0.711 & 0.17 & 0.653 & 0.653 & 0 & 0 & 0 \\
0 & 0.149 & 0.27 & 0.782 & 0.782 & 0 & 0 & 0 \\
0 & 0 & 0.782 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\end{array}
\qquad (10)
$$

For instant, considering N3 is ON and the other states remains in an OFF state.N6 in ON state,compute the influence of Maintaining social distance, wearing mask and continuous hand wash on all the other factors considered and from the simulation results we can obtain the causal strength of the concepts.

Maintaining social distance, wearing mask and continuous hand wash is in ON state then the initial state vector $\mathrm{A}(0) = (0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)$

Now

$$A(1) = \mathrm{A}(0) \times TrF(M) = (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0.17 \quad 0.27 \quad 0.782)$$

$$\Lsh (0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1) = \mathrm{A}(1) \qquad (11)$$

$$A(2) = \mathrm{A}(1) \times TrF(M) = (0 \quad 0 \quad 0.782 \quad 0 \quad 0 \quad 0.17 \quad 0.27 \quad 0.782)$$

$$\Lsh (0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1) = \mathrm{A}(2) = \mathrm{A}(1). \qquad (12)$$

The limiting cycle arrives and is observe that influence of state N3 is triggered the state N8, its well known that Maintaining social distance, wearing mask and continuous hand wash is also a part of preventive measures and a self-discipline to be followed byevery individual.

DOI: 10.5281/zenodo.4127202

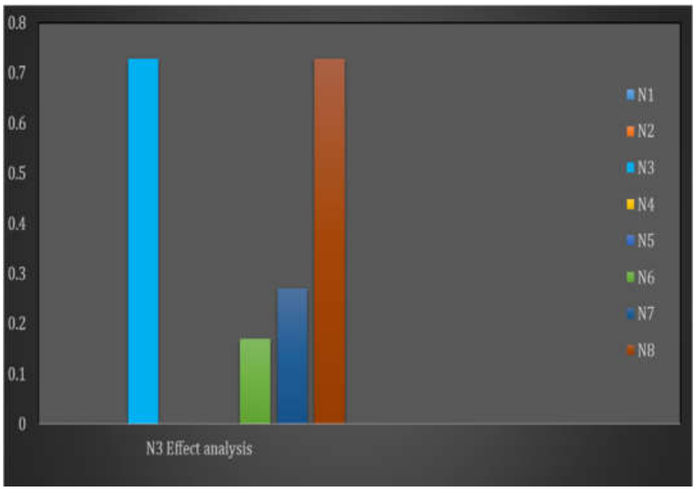


Figure-8 Influence of N3 on other concepts

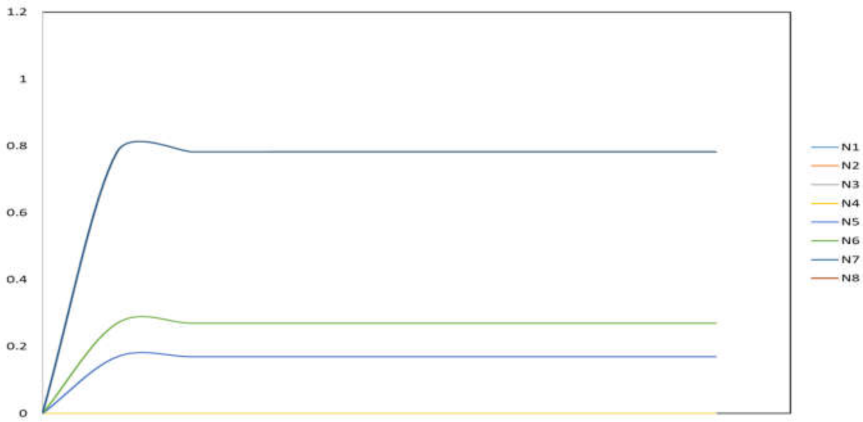


Figure-9 simulation convergence when N3 is ON

Similarly if N7 High risk factor for getting Covid-19is ON, its influence on others states is evaluated and is presented below.

High risk factor for getting Covid-19 is in ON state then the initial state vector $A(0) = (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0)$

Now

$$A(1) = \mathrm{A}(0) \times TrF(M) = (0 \quad 0.149 \quad 0.27 \quad 0.782 \quad 0.782 \quad 0 \quad 0 \quad 0)$$

$$\hookrightarrow (0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0) = A(1) \tag{13}$$

$$A(2) = \mathrm{A}(1) \times TrF(M) = (0 \quad 0.149 \quad 0.27 \quad 0.782 \quad 0.782 \quad 1.435 \quad 1.564 \quad 0)$$

$$\hookrightarrow (0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0) = A(2) \tag{14}$$

$$A(3) = \mathrm{A}(2) \times TrF(M) = (0.653 \quad 0.86 \quad 0.44 \quad 1.435 \quad 1.435 \quad 1.435 \quad 1.564 \quad 0)$$

$$\hookrightarrow (1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0) = \mathrm{A}(3) \tag{15}$$

DOI: 10.5281/zenodo.4127202

$$A(4) = \mathrm{A}(3) \times TrF(M) = (0.653 \quad 0.86 \quad 0.44 \quad 1.435 \quad 1.435 \quad 2.928 \quad 1.713 \quad 0)$$

$$\Rightarrow (1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0) = A(4) = A(3) \tag{16}$$

Hence we obtain the limiting cycle with the resultant vector$(1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0)$. The triggering pattern isN7 → N4 → N5 → N6 → N1 → N2. From the limiting cycle obtained it is easy to predict that the High risk factor for getting Covid-19 is influenced from N1, N2, N4, N5 and N6. The evaluated results are well matched with current pandemic scenariosince, getting affected to Covid-19 may be symptomatic or asymptomatic and the patients with chronic diseases like diabetics, cancer,heart diseases etc., had a major risk of getting affected to this infectious disease. In addition, travelling historyis also had a greater influence in covid-19 outbreak.

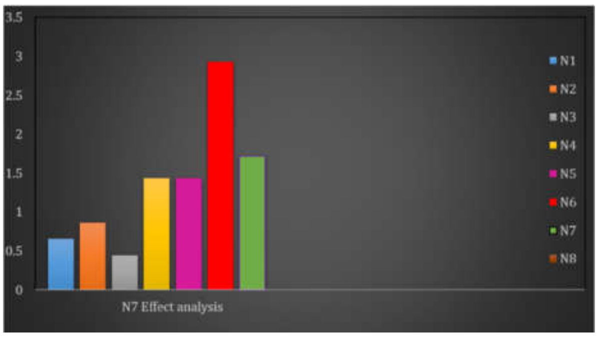


Figure-10 Influence of N7 on every concepts

Varying initial state vector by consequently stimulating each state and executing simulation process, a final state is attained after several iterations. Limiting cycle for each state is attained in this proposed methodology whose results are well matched with the real time situations.

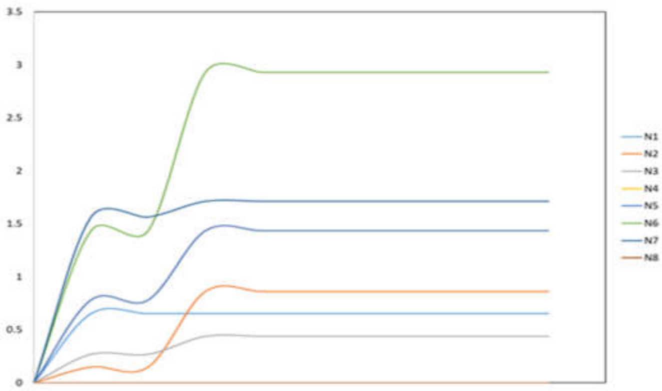


Figure-11 simulation convergence when N7 is ON

5.Comparison analysis

To show the benefits of the proposed method the comparative analysis is made between contradiction degree of the factors and that of the experts. As shown in the previous section the contradiction degree for the experts is considered and the connection matrix is combined using Plithogenic operator. To compare the methods , consider Maintaining social distance, wearing mask and continuous hand wash is in ON state then the initial state vector $\mathrm{A}(0) = (0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)$

$$A(1) = \mathrm{A}(0) \times TrF(M)$$

DOI: 10.5281/zenodo.4127202

$$A(2) = \mathrm{A}(1) \times TrF(M) = \mathrm{A}(1) \quad (17)$$

The limiting cycle arrives and is observe that influence of state N3 is triggered the state N8, its well-known that Maintaining social distance, wearing mask and continuous hand wash is also a part of preventive measures and a self-discipline to be followed by every individual.

In case of contradiction degree between the factors, Maintaining social distance, wearing mask and continuous hand wash is in ON state then the initial state vector $\mathrm{A}(0) = (0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)$ and considering the connection matrix of expert 1,

$$\mathrm{A}(1) = \mathrm{A}(0) \times \mathrm{CM}(\mathrm{E}_1) = (1 \quad 0.875 \quad 0.75 \quad 0.625 \quad 0.5 \quad 0.606 \quad 0.588 \quad 0.729)$$

$$\hookrightarrow (1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0) = A(1) \quad (18)$$

$$A(2) = \mathrm{A}(1) \times \mathrm{CM}(\mathrm{E}_1) = (1 \quad 0.875 \quad 0.75 \quad 0.625 \quad 0.5 \quad 0.606 \quad 0.588 \quad 0.729)$$

$$\hookrightarrow (1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0) = A(2) = A(1) \quad (19)$$

Limiting cycle is attained and is observed that when the N3 is ON it triggers the state N1, which is not matched with the real time situation rather the proposed methodology is concurred with the real time situation and in addition in the proposed method the opinions can be obtained from more than one expert.

6.Conclusion

The research work proposes a plithogenic approach to fuzzy cognitive map. The interpretation of contradiction degree to a plithogenic operator is used to accumulate the expert's opinion. The accumulated connection matrix is then utilized for the simulation which is based on fuzzy cognitive map.In order to assess the accuracy, the proposed model isutilised for the analysis of current pandemic situation, a covid-19 outbreak. The factors influencing the considered system and linguistic scales representing a strength between them was obtained from three distinct expert members and the simulation was done by using proposed methodology to increase the accuracy of results. The results are well matched with real time situations. This work can be further extended to Plithogenic approach to Neutrosophic Cognitive Map in the future.

Funding: "This research received no external funding"

Conflicts of Interest: "The authors declare no conflict of interest."

References

[1] Axelrod.R, "Structure of decision: Thecognitive maps of political elites"Princeton University Press,1976.

[2] Zadeh, L. A. "Fuzzy set", *Inform and Control*, 1965,v.8, 338-353.

[3] Kosko.B, "Fuzzy cognitive maps", *Int. J. Man-Machine Studies*, 1986,v. 24,no. 3, 65-75,.

[4] C. E. Pelaez, J. B. Bowles, "Applying fuzzy cognitive-maps knowledge-representation to failure modes effects analysis", *Reliability and Maintainability Symposium Proceedings*, 1995,450-456,.

[5] Papageorgiou, Elpiniki I;"A Fuzzy Inference Map approach to cope with uncertainty in modeling medical knowledge and making decisions"DOI: 10.3233/IDT-2011-0108.

[6] E. Yesil, M. F. Dodurka, L. Urbas L, "Triangular Fuzzy Number Representation of Relations in Fuzzy Cognitive Maps", *IEEE International Conference on Fuzzy Systems* (FUZZ-IEEE), DOI: 10.1109/FUZZ-IEEE.2014.6891653,2014.

[7] M. F. Dodurka, A. Sahin, E. Yesil, L. Urbas, "Learning of FCMs with links represented via fuzzy triangular numbers", *IEEE International Conference on Fuzzy Systems* (FUZZ-IEEE), 2015,1-8.

[8] F. Dodurka, E. Yesil, L. Urbas, "Causal effect analysis for fuzzy cognitive maps designed with non-singleton fuzzy numbers", *Neurocomputing* , 2017,v. 232, 122-132,.
[9] Nivetha Martin and FlorentinSmarandache,"Plithogenic Cognitive Maps in Decision Making", *International Journal of Neutrosophic Science*, 2020,v.7, no. 1, 8-31.
[10] Smarandache, F. "Plithogeny, Plithogenic Set, Logic, Probability, and Statistics",arXiv:1808.03948,2018.
[11] Mohamed Abdel-Basset, Rehab Mohamed, Abd El-Nasser H. Zaied and FlorentinSmarandache,"A hybrid plithogenic decision-making approach with quality function deployment for selecting supply chain sustainability metrics" *Symmetry*, 2019, v.11, 903.
[12] Mohamed Abdel-Basseta, Mohamed El-hosenyb, AbduallahGamala, FlorentinSmarandachec, "A novel model for evaluation Hospital medical care systems based on plithogenic sets",https://doi.org/10.1016/j.artmed.2019.101710,2019.
[13] Grida, Mohamed; Rehab Mohamed; and Abdelnaser H. Zaid. "A Novel Plithogenic MCDM Framework for Evaluating the Performance of IoT Based Supply Chain." Neutrosophic Sets and Systems,2020, v.33,no. 1.
[14] Abdel-Basset M, Mohamed R, "A novel plithogenic TOPSIS- CRITIC model for sustainable supply chain risk management", *Journal of Cleaner Production*, doi: https:// doi.org/10.1016/j.jclepro.2019.119586.
[15] Zimmermann H.J (2011) Fuzzy Set Theory and its Application,Fourth ed., Springer.
[16] FlorentinSmarandache ,Vasanthakandasamy W.B, (2003) "Fuzzy Cognitive Maps and Neutrosophic Cognitive Maps", Xiquan, Phoenix.
[17] GrouposP.Peter,AnninouP.Antigoni,GrouposP.Vasileios,"A New Mathematical Modelling Approach for Viticulture and Winemaking Using Fuzzy Cognitive Maps",*IFAC-PapersOnLine*,2015,v. 48,no.24, 15-20.
[18] Shing-ChungNgan,"Decision making with extended fuzzy linguistic computing, with applications to new product development and survey analysis",https://doi.org/10.1016/j.eswa.2011.04.213,2011.
[19] Ying-Ming Wang, Jian-Bo Yang, Dong-Ling Xu, Kwai-Sang Chin, "On the centroids of fuzzy numbers", *Fuzzy Sets Syst*., 2006, v.157, 919 – 926,.
[20] Amirkani,A. ;Papageorgiou,E.I.;Mohseni,A.;Mosavi,M.R."A Review of Fuzzy Cognitive Maps in Medicine: Taxonomy, Methods, and Applications", *Computer Methods and Programs in Biomedicine*,2017,v.142,129-145.
[21] Stylios C.D., Georgopoulos V.C. ,"Fuzzy Cognitive Maps Structure for Medical Decision Support Systems", In: Forging New Frontiers: Fuzzy Pioneers II. Studies in Fuzziness and Soft Computing, Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-540-73185-6_7,2008,v. 218.
[22] Douali,N. ; Papageorgiou,E.I. ; Roo, J.D. ; Cools,H. ; Jaulent,M.C. Clinical Decision Support System based on Fuzzy Cognitive Maps. *Journal of Computer Science & Systems Biology*,2015,v.8,no.1, 112-120.
[23] Khodadadi,M., ; Shayanfar,H. ; Maghooli,K. ; HooshangMazinan,A."Prediction of stroke probability occurrence based on fuzzy cognitive maps", AUTOMATIKA,2019,v.4, 385-392.
[24] Papageorgiou,E.I.;Jayashree,S. ;Akila, K.;Nikolaos,P.A, "Risk Management Model for Familial Breast Cancer: A New Application using Fuzzy Cognitive Map method",*Computer methods and programs in biomedicine*,2015,v.122,no.2, 1-13.
[25] Neil ,A.,Hilah,S."Fuzzy cognitive mapping: An old tool with new uses in nursing research", *Journal of Advanced Nursing*,2019,v.75.no.12,1-8.
[26] Merlyn Margaret,H. ;Lilly Merline,W.A. "Study on the symptoms of migraine using induced fuzzy cognitive maps",*AIP Conference Proceedings*,2019,v.2112, 1-5.
[27] N. Chen, M. Zhou, X. Dong, J. Qu, F. Gong, Y. Han, Y. Qiu, J. Wang, Y. Liu, Y. Wei, J. Xia, T. Yu, X. Zhang, L. Zhang, "Epidemiological and clinical characteristics of 99 cases of 2019 novel coronavirus pneumonia in Wuhan, China: A descriptive study", *THE LANCET*, doi:10.1016/S0140-6736(20)30211-7 Medline,395,2020, 507–513.
[28] P. Zhou, X.-L. Yang, X.-G. Wang, B. Hu, L. Zhang, W. Zhang, H.-R. Si, Y. Zhu, B. Li, C.- L. Huang, H.-D. Chen, J. Chen, Y. Luo, H. Guo, R.-D. Jiang, M.-Q. Liu, Y. Chen, X.-R. Shen, X. Wang, X.-S. Zheng, K. Zhao, Q.-J. Chen, F. Deng, L.-L. Liu, B. Yan, F.-X. Zhan, Y.-Y. Wang, G.-F. Xiao, Z.-L. Shi, "A

DOI: 10.5281/zenodo.4127202

pneumonia outbreak associated with a new coronavirus of probable bat origin", Nature, doi:10.1038/s41586-020-2012-7 Medline,2020,v.579, 270–273.

[29] Huang C, Wang Y, Li X, Ren L, Zhao J, Hu Y, et al. "Articles Clinical features of patients infected with 2019 novel coronavirus in Wuhan. China 2020",*THE LANCET*,http://dx.doi.org/10.1016/S0140-6736(20)30183-5,2020,v.395, 497-506.

[30] Wang D, Hu B, Hu C, Zhu F, LiuX, Zhang J, et al. "Clinical characteristics of 138 hospitalized patients with 2019 novel coronavirus–Infected pneumonia in Wuhan, China",JAMA. 2020;v.323,no.11, 1061-1069, doi:10.1001/jama.2020.1585.

[31] Wu Z, McGoogan JM.,"Characteristics of and Important Lessons from the Coronavirus Disease 2019 (COVID-19) Outbreak in China: Summary of a Report of 72 314 Cases from the Chinese Center for Disease Control and Prevention", JAMA 2020, http://dx.doi.org/10.1001/jama.2020.2648.

[32] Prem Kumar Singh, "Multi–granular based n–valued neutrosophic contexts analysis", *Granular Computing*, 2020,v.5, 287-301. DOI: 10.1007/s41066-019-00160-y.

[33] Prem Kumar Singh, "Plithogenic set for multi-variable data analysis",*International Journal of Neutrosophic Sciences*,2020, v. 1, no. 2, 81-89, DOI: 10.5281/zenodo.3689808.

[34] Prem Kumar Singh, "Three-way n-valued neutrosophic concept lattice at different granulation",*International Journal of Machine Learning and Cybernetics*, 2018,v. 9, no. 11,1839-1855.

[35] F. Smarandache, "Extension of Soft Set to Hypersoft Set, and then to Plithogenic Hypersoft Set", *Neutrosophic sets and system*, 2018,v. 22,168-170.

A New Multi-Attribute Decision Making Method with Single-Valued Neutrosophic Graphs

Juanjuan Ding [1], Wenhui Bai [1] and Chao Zhang [1,2,*]

[1] School of Computer and Information Technology, Shanxi University, Taiyuan 030006, Shanxi, China
[2] Key Laboratory of Computational Intelligence and Chinese Information Processing of Ministry of Education, Shanxi University, Taiyuan 030006, Shanxi, China

* Correspondence: czhang@sxu.edu.cn

Abstract

In most realistic situations, the theory and method of multi-attribute decision-making have been widely used in different fields, such as engineering, economy, management, military, and others. Although many studies in some extended fuzzy contexts have been explored with multi-attribute decision-making, it is widely recognized that single-valued neutrosophic sets can describe incomplete, indeterminate and inconsistent information more easier. In this paper, aiming at addressing multi-attribute decision-making problems with single-valued neutrosophic information, related models and multi-attribute decision-making approaches based on the fuzzy graph theory are studied. In specific, the notion of single-valued neutrosophic sets and graphs is firstly introduced together with several common operational laws. Then a multi-attribute decision making method based on single-valued neutrosophic graphs is established. Finally, an illustrative example and a comparative analysis are conducted to verify the feasibility and efficiency of the proposed method.

Keywords: single-valued neutrosophic sets; multi-attribute decision-making; fuzzy graph theory; single-valued neutrosophic graphs

1.Introduction

As an important part of modern decision-making sciences, multi-attribute decision-making aims to make decision analysis of a limited number of options from the perspective of multiple attributes, and then choose an optimal choice of alternatives by means of information integration rules. Its theories and methods have been widely used in many areas, and it has effectively promoted the development of social economy [1]. Due to the limitation of decision makers' cognitive limitations and the increasing complexity of decision-making problems, it is difficult for decision makers to process decision-making information accurately. Since Zadeh put forward the fuzzy set theory [2], fuzzy multi-attribute decision-making has become an important research direction. With the progress of fuzzy sets, a variety of generalized fuzzy sets have been proposed. Among them, Atanassov's intuitionistic fuzzy sets [3] are more flexible and practical than traditional fuzzy sets in addressing uncertainties. Further, Smarandache [4] proposed the concept of neutrosophic sets. A neutrosophic set contains three types of membership functions (truth ones, indeterminacy ones and falsity ones). Afterwards, single-valued neutrosophic sets were introduced by

DOI: 10.5281/zenodo.4129304

Received: July 07, 2020 Accepted: Octobre 09, 2020

Smarandache [4-5] and Wang et al. [6], which can be seen as a generalized form of intuitionistic fuzzy sets and have numerous applications in real-life applications. At present, single-valued neutrosophic multi-attribute problems have been widely studied by scholars and practitioners [7-9].

Rosenfeld [10] proposed the notion of fuzzy graphs. Later on, Mordeson and Peng [11] provided several common operations of fuzzy graphs. Ye [12] gave the idea of single-valued neutrosophic minimum spanning trees and introduced a corresponding clustering method. Yang et al. [13] elaborated single-valued neutrosophic relations. Dhavaseelan et al. [14] defined strong neutrosophic graphs. Akram and Shahzadi [15] introduced the notion of neutrosophic soft graphs in 2016. Recently, Akram and Shahzadi [16] studied properties of single-valued neutrosophic graphs by level graphs. They also developed some operations according to presented novel notions in Broumi et al. [17] and Shah-Hussain [18]. Akram and Sitara [19] studied single-valued neutrosophic graph structures for decision-making issues. By taking advantages of diverse neutrosophic graphs, Broumi et al. [20] explored panning tree problems with neutrosophic edge weights. Then Broumi et al. [21] further studied a shortest path problem by virtue of the Bellman algorithm in the neutrosophic context, which can lay a solid foundation on the neutrosophic graph theory. Moreover, Karaaslan and Davvaz [22] discussed several novel notions of single-valued neutrosophic graphs along with an application in decision-making. Karaaslan [23] explored a new Gaussian single-valued neutrosophic numbers and then presented corresponding applications in multi-attribute decision-making.

In this paper, the concept, operators and score functions of single-valued neutrosophic sets and graphs are firstly reviewed, which are referred to single-valued neutrosophic graph structures. Then a multi-attribute decision rule by means of single-valued neutrosophic graphs is proposed. Finally, an illustrative example and a comparative analysis are conducted to verify the feasibility and efficiency of the proposed method.

2. Basic knowledge

2.1 Single-valued neutrosophic sets

Definition 1 [4] Let X be a universe of discourse. A single-valued neutrosophic set A on X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. Then a single-valued neutrosophic set A on X is provided by:

$$A=\left\{x\left(T_A(x),I_A(x),F_A(x)\right)\middle|x\in X\right\},$$

where $T_A(x),I_A(x),F_A(x)\to \operatorname{int}[0,1]$ for all $x\in X$ to the set A, then the following condition is true: $0\le T_A(x)+I_A(x)+F_A(x)\le 3$. In addition, all single-valued neutrosophic sets on X are denoted by $SVN(X)$.

Definition 2 [5] Let X be the universe of discourse. $\forall A,B\in SVN(X)$, then the following operations are defined as follows:

1. $A\oplus B=\left\{\left\langle x,T_A(x)+T_B(x)-T_A(x)T_B(x),I_A(x)I_B(x),F_A(x)F_B(x)\right\rangle\right\}$;

2. $A\otimes B=\left\{\left\langle x,T_A(x)T_B(x),I_A(x)+I_B(x)-I_A(x)I_B(x),\right.\right.$

$$F_A(x)+F_B(x)-F_A(x)F_B(x)\rangle\};$$

3. $\lambda\cdot A=\left\{\left\langle x,1-\left(1-T_A(x)^{\lambda}\right),I_A(x)^{\lambda},F_A(x)^{\lambda}\right\rangle\right\}$;

4. $A^{\lambda}=\left\{\left\langle x,T_A(x)^{\lambda},1-\left(1-I_A(x)^{\lambda}\right),1-\left(1-F_A(x)^{\lambda}\right)\right\rangle\right\}$;

5. $A\boxminus B=\left\{\left\langle x,\dfrac{T_A(x)-T_B(x)}{1-T_B(x)},\dfrac{I_A(x)}{I_B(x)},\dfrac{F_A(x)}{F_B(x)}\right\rangle\right\}$;

6. $A\boxdot B=\left\{\left\langle x,\dfrac{T_A(x)}{T_B(x)},\dfrac{I_A(x)-I_B(x)}{1-I_B(x)},\dfrac{F_A(x)-F_B(x)}{1-F_B(x)}\right\rangle\right\}$;

7. $A^{c}=\left\{\left\langle x,F_A(x),1-I_A(x),T_A(x)\right\rangle\right\}$;

8. $A\cap B=\left\{\left\langle x,T_A(x)\wedge T_B(x),I_A(x)\vee I_B(x),F_A(x)\vee F_B(x)\right\rangle\right\}$;

9. $A\cup B=\left\{\left\langle x,T_A(x)\vee T_B(x),I_A(x)\wedge I_B(x),F_A(x)\wedge F_B(x)\right\rangle\right\}$.

Definition 3 [6] Suppose $x=(T,I,F)$ is a single-valued neutrosophic number. A score function with regard to x is provided as the following mathematical expression:

$$s(x)=T+1-I+1-F.$$

2.2 Single-valued neutrosophic graphs

Definition 4 [19] Suppose V is a finite universe of discourse. A single-valued neutrosophic graph G is defined as the following form:

$$G=(C,D),$$

where C is a single-valued neutrosophic set on V with $T_C,I_C,F_C:V\rightarrow \mathrm{int}[0,1]$, D is a single-valued neutrosophic set on $V\times V$ with $T_C,I_C,F_C:V\times V\rightarrow \mathrm{int}[0,1]$. For any a_x and a_y, the followings are true:

$$T_D(a_xa_y)\le \min\left(T_C(a_x),T_C(a_y)\right),$$

$$I_D(a_xa_y)\le \max\left(I_C(a_x),I_C(a_y)\right),$$

$$F_D\left(a_x a_y\right) \le \max\left(F_C\left(a_x\right), F_C\left(a_y\right)\right).$$

Then $G=(C,D)$ is a single-valued neutrosophic graph of $G^*=(V,E)$. In specific, C is single-valued neutrosophic vertices on G, and D is single-valued neutrosophic edges on G.

3. Multi-attribute decision making based on single-valued neutrosophic graphs

In this section, for single-valued neutrosophic multi-attribute decision making problems with correlations and priorities among attributes, a multi-attribute decision making approach based on single-valued neutrosophic graphs is established. Firstly, the basic model of the problem is described. After that, a single-valued neutrosophic multi-attribute decision making algorithm with correlations and priorities is given.

3.1. The model building

In the problem of single-valued neutrosophic multi-attribute decision making with correlations and priorities among attributes at the same time, let a set of alternatives be $Q=\{q_1,q_2,\cdots,q_m\}$, a set of attributes be $V=\{a_1,a_2,\cdots,a_n\}$, and a set of attribute weights be $W=\left(w_1,w_2,\cdots,w_n\right)^T$ that satisfies $w_j\in[0,1]$ and $\sum_{j=1}^{n} w_j=1$. In addition, there is a linear priority relationship $a_1 \succ a_2 \succ \cdots \succ a_n$ among all attributes, where $a_1 \succ a_2$ refers to a_1 is more important than a_2. Then, a decision maker can evaluate an alternative $q_k\left(k=1,2,\cdots,m\right)$ by using the attribute $a_j\left(j=1,2,\cdots,n\right)$, and the evaluation result is given in the form of single-valued neutrosophic numbers, thus a single-valued neutrosophic decision making matrix $D=\left(d_{kj}\right)_{m\times n}$ is formed. In order to solve the above-stated problem by means of advantages of single-valued neutrosophic graphs, let C be a single-valued neutrosophic set on V with $T_C,I_C,F_C:V\to \operatorname{int}[0,1]$, and D be a single-valued neutrosophic set on $V\times V$ with $T_D,I_D,F_D:V\to \operatorname{int}[0,1]$. For any a_i and a_j, the followings are true:

$$T_D\left(a_i a_j\right) \le \min\left(T_C\left(a_i\right), T_C\left(a_j\right)\right),$$

$$I_D\left(a_i a_j\right) \le \max\left(I_C\left(a_i\right), I_C\left(a_j\right)\right),$$

$$F_D\left(a_i a_j\right) \le \max\left(F_C\left(a_i\right), F_C\left(a_j\right)\right),$$

thus a single-valued neutrosophic graph $G=(C,D)$ of graph $G^*=(V,E)$ is built. At last, after integrating several single-valued neutrosophic information under multiple attributes, overall attribute values that correspond to alternatives $q_1,q_2,\cdots,q_m$ are analyzed and an optimal alternative is selected. On the whole, the above multi-attribute decision making process includes representations, constructions, analysis and other stages of decision making information.

3.2 The model algorithm

In what follows, a single-valued neutrosophic multi-attribute decision making algorithm with correlations and priority relationships is given. The core of this algorithm lies in handling the relevance and priority relationships among attributes effectively.

Stage 1: In terms of dealing with correlations among attributes, an energy coefficient in the background of single-valued neutrosophic sets is developed for solving the degree of interactions among attributes. For two related attributes a_i and $a_j\left(i, j=1,2,\cdots,n\right)$, the energy coefficient of single-valued neutrosophic information is expressed as follows:

$$\varphi_{ij}=\left\langle \sum_{\alpha\in T_D\left(a_i a_j\right)}\alpha^2, \sum_{\eta\in I_D\left(a_i a_j\right)}\eta^2, \sum_{\beta\in F_D\left(a_i a_j\right)}\beta^2 \right\rangle,$$

it is easy to see if $i=j$, then $\varphi_{ij}=\varphi_{ji}$ is true. At this time, the energy coefficient of the single-valued neutrosophic information reaches the maximum value $\langle 1,0,0\rangle$; if all attributes are independent of each other, the energy coefficient of single-valued neutrosophic information reaches the minimum value $\langle 0,0,1\rangle$ at this time. In most cases, the single-valued neutrosophic information energy coefficient is presented between $\langle 0,0,1\rangle$ and $\langle 1,0,0\rangle$.

Stage 2: In terms of solving the priority relationship, to obtain attribute weights objectively, a concept of eccentricities is developed in the background of single-valued neutrosophic graphs, and the idea of linear priority relations among attributes is integrated. At first, the eccentricity of every single-valued neutrosophic vertices is calculated correspondingly. Suppose $G=\left(C,D\right)$ is a single-valued neutrosophic graph of $G^*=\left(V,E\right)$, and there are $n+1$ single-valued neutrosophic vertices $u=a_0,a_1,\cdots,a_{n-1},a_n=v$. Then, the single-valued neutrosophic eccentricity for every attribute of V is $e\left(a_j\right)=\left\langle \max_{x\in V}\left\{F_{\mathrm{D}}\left(a_i a_j\right)\right\},\min_{x\in V}\left\{1-I_{\mathrm{D}}\left(a_i a_j\right)\right\},\min_{x\in V}\left\{T_{\mathrm{D}}\left(a_i a_j\right)\right\}\right\rangle$ $\left(j=1,2,\cdots,n\right)$. If there is a liner priority relationship among attributes $a_1\succ a_2\succ\cdots\succ a_n$, then the attribute weight w_j can be derived from the above single-valued neutrosophic eccentricity, thus $w_j=\prod_{k=1}^{j}S_{k-1}$ can be obtained, where $S_j=e\left(a_j\right)$ when $j\neq 0$ and $S_j=\langle 1,0,0\rangle$ when $j=0$. Therefore, when all attribute weights are completely unknown, attribute weights can be obtained by using the priority relationship between attributes.

At last, based on the above idea of coping with correlations and priority relationships among attributes, a single-valued neutrosophic multi-attribute decision making algorithm is eventually established that is shown as follows.

Input A single-valued neutrosophic decision making matrix $D=\left(d_{kj}\right)_{m\times n}$, the linear priority relationship among attributes $a_1\succ a_2\succ\cdots\succ a_n$, and a single-valued neutrosophic graph $G=\left(C,D\right)$ that describes the correlation among attributes.

Output The best alternative.

Step 1 Calculate the energy coefficient φ_{ij} of single-valued neutrosophic information between two related attributes a_i and a_j.

Step 2 Calculate the attribute weight w_j by using the priority relationship among attributes.

Step 3 Calculate the overall attribute value $\tilde{q}_k = \sum_{j=1}^{n} w_j \left(\sum_{x=1}^{n} d_{kx}\varphi_{xj} \right)$ of an alternative $q_k \left(k = 1, 2, \cdots, m\right)$.

Step 4 Calculate the score function $s\left(\tilde{q}_k\right)$ that corresponds to the overall attribute value of an alternative q_k.

Step 5 Determine the best alternative $q^* = \max_{k=1}^{m} \left\{ s\left(\tilde{q}_k\right) \right\}$.

4. An illustrative example

This section intends to present detailed processes of addressing a single-valued neutrosophic multi-attribute decision making problem by utilizing the case study from Literature [24], then a comparative analysis is arranged to demonstrate the effectiveness of the proposed single-valued neutrosophic multi-attribute decision making algorithm.

4.1 Case descriptions

Indian government had been issued a global tender to select the contractor for these projects in the newspaper and considered five attributes required, i.e., Technology Expertise$\left(a_1\right)$, Service quality$\left(a_2\right)$, Bandwidth$\left(a_3\right)$, Internet speed $\left(a_4\right)$ and Customer Services $\left(a_5\right)$, and the importance of attributes is set by $W = \left\{w_1, w_2, w_3, w_4, w_5,\right\}^T$. In addition, suppose there is a priority relationship $a_1 \succ a_2 \succ a_3 \succ a_4 \succ a_5$ among these five attributes, that is, the person responsible for business gives the highest priority to technology expertise, then other four attributes are concerned by the same individual in succession. The five contractors (i.e., alternatives) namely, "Jaihind Road Builders private (Pvt.) limited (Ltd.) "$\left(q_1\right)$, "J.K. Construction" $\left(q_2\right)$, "Build quick Infrastructure Pvt. Ltd."$\left(q_3\right)$,"Relcon Infra projects Ltd." $\left(q_4\right)$, and" Tata Infrastructure Ltd." $\left(q_5\right)$ bid for these projects. In order to reasonably describe the incompleteness in the above-stated MADM problem, a single-valued neutrosophic decision making matrix $D = \left(d_{kj}\right)_{5\times5}$ is presented in Table 1. Afterwards, a single-valued neutrosophic graph $G = \left(C, D\right)$ of $G^* = \left(V, E\right)$ is established to describe correlations between attributes, where $E = \left\{a_1a_2, a_1a_3, a_1a_4, a_1a_5, a_2a_3, a_2a_4, a_2a_5, a_3a_4, a_3a_5, a_4a_5\right\}$, and the established single-valued neutrosophic graph is presented in the following Figure 1. In light of the above expressions, the proposed single-valued neutrosophic multi-attribute decision making algorithm by virtue of single-valued neutrosophic graphs will be used to obtain the best contractor for Indian government.

4.2 Processes of single-valued neutrosophic multi-attribute decision making

According to the proposed single-valued neutrosophic multi-attribute decision making algorithm, the energy coefficient of the single-valued neutrosophic information between attributes is firstly calculated as follows.

$$\varphi_{12} = \left\langle \sum_{\alpha \in T_D(a_i a_j)} \alpha^2, \sum_{\eta \in I_D(a_i a_j)} \eta^2, \sum_{\beta \in F_D(a_i a_j)} \beta^2 \right\rangle = \langle 0.16, 0.09, 0.16 \rangle.$$

Similarity, it is not difficult to get

$\varphi_{13} = \langle 0.04, 0.25, 0.36 \rangle$, $\varphi_{14} = \langle 0.09, 0.16, 0.36 \rangle$, $\varphi_{15} = \langle 0.09, 0.25, 0.16 \rangle$,
$\varphi_{23} = \langle 0.04, 0.25, 0.25 \rangle$, $\varphi_{24} = \langle 0.09, 0.16, 0.36 \rangle$, $\varphi_{25} = \langle 0.09, 0.25, 0.16 \rangle$,
$\varphi_{34} = \langle 0.04, 0.25, 0.16 \rangle$, $\varphi_{35} = \langle 0.04, 0.25, 0.36 \rangle$, $\varphi_{45} = \langle 0.09, 0.25, 0.36 \rangle$.

Table 1. The single-valued neutrosophic decision making matrix

	a_1	a_2	a_3	a_4	a_5
q_1	$\langle 0.5, 0.3, 0.4 \rangle$	$\langle 0.5, 0.2, 0.3 \rangle$	$\langle 0.2, 0.2, 0.6 \rangle$	$\langle 0.3, 0.2, 0.4 \rangle$	$\langle 0.3, 0.3, 0.4 \rangle$
q_2	$\langle 0.7, 0.1, 0.3 \rangle$	$\langle 0.6, 0.2, 0.3 \rangle$	$\langle 0.6, 0.3, 0.2 \rangle$	$\langle 0.6, 0.4, 0.2 \rangle$	$\langle 0.7, 0.1, 0.2 \rangle$
q_1	$\langle 0.5, 0.3, 0.4 \rangle$	$\langle 0.6, 0.2, 0.4 \rangle$	$\langle 0.6, 0.1, 0.2 \rangle$	$\langle 0.5, 0.1, 0.3 \rangle$	$\langle 0.6, 0.4, 0.3 \rangle$
q_1	$\langle 0.7, 0.3, 0.2 \rangle$	$\langle 0.7, 0.2, 0.2 \rangle$	$\langle 0.4, 0.5, 0.2 \rangle$	$\langle 0.5, 0.2, 0.2 \rangle$	$\langle 0.4, 0.5, 0.4 \rangle$
q_1	$\langle 0.4, 0.1, 0.3 \rangle$	$\langle 0.5, 0.1, 0.2 \rangle$	$\langle 0.4, 0.1, 0.5 \rangle$	$\langle 0.4, 0.3, 0.6 \rangle$	$\langle 0.3, 0.2, 0.4 \rangle$

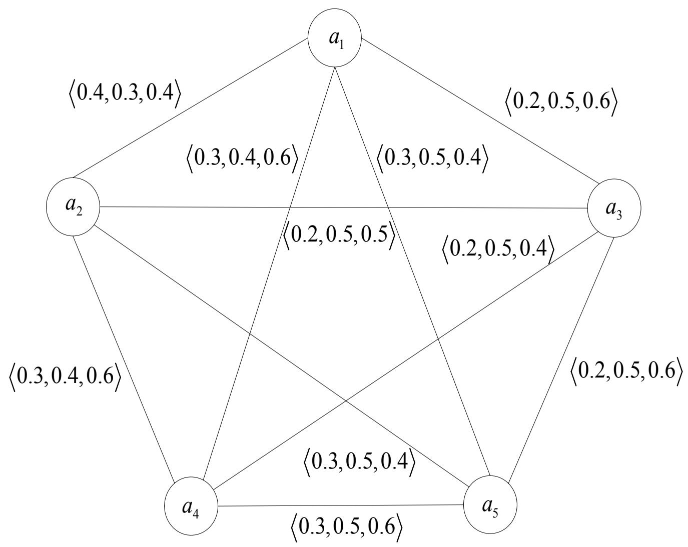


Figure 1: The single-valued neutrosophic graph for describing correlations between attributes

Then, attribute weights by virtue of the linear priority relationship $a_1 \succ a_2 \succ a_3 \succ a_4 \succ a_5$ are calculated, and the following single-valued neutrosophic eccentricities for every attributes are obtained.

$e(a_1) = \langle 0.6, 0.5, 0.2 \rangle$, $e(a_2) = \langle 0.6, 0.5, 0.2 \rangle$, $e(a_3) = \langle 0.6, 0.5, 0.2 \rangle$, $e(a_4) = \langle 0.6, 0.5, 0.2 \rangle$,
$e(a_5) = \langle 0.6, 0.5, 0.2 \rangle$.

In light of the above results, $S_0=\langle 1,0,0\rangle$ is further obtained. In a similar manner, $S_1=\langle 0.6,0.5,0.2\rangle$, $S_2=\langle 0.6,0.5,0.2\rangle$, $S_3=\langle 0.6,0.5,0.2\rangle$, $S_4=\langle 0.6,0.5,0.2\rangle$, and $S_5=\langle 0.6,0.5,0.2\rangle$. Hence, by virtue of $w_j=\prod_{k=1}^{j} S_{k-1}$, where $S_j=e(a_j)$ when $j\neq 0$ and $S_j=\langle 1,0,0\rangle$ when $j=0$, the attribute weight can be obtained via the above results ($e(a_j)$ is obtained in the above process): $w_1=\langle 1,0,0\rangle$, $w_2=\langle 0.6,0.5,0.2\rangle$, $w_3=\langle 0.36,0.75,0.36\rangle$, $w_4=\langle 0.216,0.875,0.488\rangle$, $w_5=\langle 0.1296,0.9375,0.5904\rangle$.

Afterwards, the following overall attribute values are calculated.

$$\begin{aligned}\widetilde{q_1}=\ & w_1\otimes\left(d_{11}\otimes\varphi_{11}\oplus d_{12}\otimes\varphi_{21}\oplus d_{13}\otimes\varphi_{31}\oplus d_{14}\otimes\varphi_{41}\oplus d_{15}\otimes\varphi_{51}\right)\oplus\\ & w_2\otimes\left(d_{11}\otimes\varphi_{12}\oplus d_{12}\otimes\varphi_{22}\oplus d_{13}\otimes\varphi_{32}\oplus d_{14}\otimes\varphi_{42}\oplus d_{15}\otimes\varphi_{52}\right)\oplus\\ & w_3\otimes\left(d_{11}\otimes\varphi_{13}\oplus d_{12}\otimes\varphi_{23}\oplus d_{13}\otimes\varphi_{33}\oplus d_{14}\otimes\varphi_{43}\oplus d_{15}\otimes\varphi_{53}\right)\oplus\\ & w_4\otimes\left(d_{11}\otimes\varphi_{14}\oplus d_{12}\otimes\varphi_{24}\oplus d_{13}\otimes\varphi_{34}\oplus d_{14}\otimes\varphi_{44}\oplus d_{15}\otimes\varphi_{54}\right)\oplus\\ & w_5\otimes\left(d_{11}\otimes\varphi_{15}\oplus d_{12}\otimes\varphi_{25}\oplus d_{13}\otimes\varphi_{35}\oplus d_{14}\otimes\varphi_{45}\oplus d_{15}\otimes\varphi_{55}\right)=\langle 0.7741,0.0017,0.001\rangle.\end{aligned}$$

Similarly, it is not difficult to get

$$\widetilde{q_2}=\langle 0.9266,0.0007,0.0002\rangle,\ \widetilde{q_3}=\langle 0.8539,0.0011,0.0005\rangle,$$
$$\widetilde{q_4}=\langle 0.9102,0.0033,0.0002\rangle,\ \widetilde{q_5}=\langle 0.7587,0.0003,0.0007\rangle.$$

At last, the score function $s(\widetilde{q_z})$ can be calculated as $s(\widetilde{q_1})=2.7714$, $s(\widetilde{q_2})=2.9257$, $s(\widetilde{q_3})=2.8523$, $s(\widetilde{q_4})=2.9067$, $s(\widetilde{q_5})=2.7577$. According to the order of $s(\widetilde{q_z})$ from large to small, the ranking of emerging technology enterprise is obtained as $q_2\succ q_4\succ q_3\succ q_1\succ q_5$, so the best contractor is q_2.

4.3 Comparative analysis

For the sake of presenting validity and effectiveness of the constructed single-valued neutrosophic multi-attribute decision making algorithm, a comparative analysis is studied by using single-valued neutrosophic multi-attribute decision making approaches proposed in Literature [24], both specific calculation processes and discussions are presented below.

According to the method of logarithmic single-valued neutrosophic weighted average (L-SVNWA) operators and logarithmic single-valued neutrosophic weighted geometric (L-SVNWG) operators, a comparison analysis by using them in the above presented case study will be made. Suppose a single-valued neutrosophic making decision matrix is $D=\left(d_{kj}\right)_{m\times n}\left(k=1,2,\cdots,m, j=1,2,\cdots,n\right)$, then the following L-SVNWA operators and L-SVNWG operators are presented.

(1) The L-SVNWA operator is provided as follows:

$$\tilde{q}_k = L - SVNWA\left(\tilde{q}_{k1}, \tilde{q}_{k2}, \cdots, \tilde{q}_{kn}\right)$$
$$= \left\langle 1 - \prod_{j=1}^{n}\left(\log_{\lambda_{kj}} \alpha_{kj}\right)^{w_j}, \prod_{j=1}^{n}\left(\log_{\lambda_{kj}}\left(1 - \eta_{kj}\right)\right)^{w_j}, \prod_{j=1}^{n}\left(\log_{\lambda_{kj}}\left(1 - \beta_{kj}\right)\right)^{w_j} \right\rangle.$$

(2) The L-SVNWG operator is provided as follows:

$$\tilde{q}_k = L - SVNWA\left(\tilde{q}_{k1}, \tilde{q}_{k2}, \cdots, \tilde{q}_{kn}\right)$$
$$= \left\langle \prod_{j=1}^{n}\left(\log_{\lambda_{kj}} \alpha_{kj}\right)^{w_j}, 1 - \prod_{j=1}^{n}\left(\log_{\lambda_{kj}}\left(1 - \eta_{kj}\right)\right)^{w_j}, 1 - \prod_{j=1}^{n}\left(\log_{\lambda_{kj}}\left(1 - \beta_{kj}\right)\right)^{w_j} \right\rangle.$$

Compared with previous comparative analysis, the single-valued neutrosophic numbers of each contractor under corresponding attributes are integrated and the score of single-valued neutrosophic numbers after integrations is obtained. The contractor with larger score value is the best candidate for J.K. construction. By using L-SVNWA, the ranking of contractors is $q_2 \succ q_4 \succ q_3 \succ q_5 \succ q_1$, which is not completely consistent with the results obtained by using the method proposed in this paper. It is easy to see that there are differences in the ranking between q_5 and q_1. However, the above ranking results do not affect the selection of the best contractor, the best contractor is still the contractor q_2. By using L-SVNWG, the ranking of contractors is $q_2 \succ q_4 \succ q_3 \succ q_1 \succ q_5$, the results by using the method proposed in this paper are consistent.

Compared with the decision results obtained from the method proposed in Literature [20], the advantages of the constructed single-valued neutrosophic multi-attribute decision making algorithm are mainly reflected as follows:

(1) The constructed single-valued neutrosophic multi-attribute decision making algorithm utilizes the framework of fuzzy graphs to address multi-attribute decision making, which makes single-valued neutrosophic graphs excel in expressing correlations between attributes via edges between vertices in single-valued neutrosophic information systems, thus the addressing of correlational single-valued neutrosophic multi-attribute decision making becomes more efficient.

(2) In light of the proposed theoretical aspects of single-valued neutrosophic graphs, the constructed single-valued neutrosophic single-valued neutrosophic algorithm provides a two-stage problem solving approach by integrating the strategies with correlations and prioritization relationships at the same time, which is beneficial for completing a complicated multi-attribute decision making with high qualities.

5. Conclusions

In real world, single-valued neutrosophic sets have many advantages in dealing with uncertainties compared to fuzzy sets and intuitionistic fuzzy sets, it is easy to see that single-valued neutrosophic sets play important part in information depictions in single-valued neutrosophic, thus it is necessary to establish efficient information analysis tools for single-valued neutrosophic in single-valued neutrosophic information systems. In this paper, the concept and operation rules of single-value neutrosophic sets and graphs are firstly introduced. Then a single-valued neutrosophic algorithm is proposed based on single-valued neutrosophic graphs. Finally, a practical case study and a corresponding comparative analysis are conducted to show the applicability and effectiveness of the presented single-valued neutrosophic single-valued neutrosophic algorithm.

In terms of future works, it is noted that there still exist some interesting topics that are worth exploring. First, discussing more theoretical issues for single-valued neutrosophic graphs is necessary, such as hypergraph structures and vague hypergraph structures [25]. Second, it is meaningful to further extend single-valued neutrosophic graphs to more realistic decision making contexts, such as incomplete information systems, hybrid information systems, dynamic information systems, etc [26-28].

Funding: "The work was supported by the Key R&D program of Shanxi Province (International Cooperation, 201903D421041), the Natural Science Foundation of Shanxi (Nos. 201801D221175, 201901D211176 and 201901D211414), Training Program for Young Scientific Researchers of Higher Education Institutions in Shanxi, Research Project Supported by Shanxi Scholarship Council of China, Cultivate Scientific Research Excellence Programs of Higher Education Institutions in Shanxi (CSREP) (2019SK036), Scientific and Technological Innovation Programs of Higher Education Institutions in Shanxi (STIP) (Nos. 201802014, 2019L0066, 2019L0500), and Industry-University-Research Collaboration Pro gram Between Shanxi University and Xiaodian District."

Conflicts of Interest: "The authors declare no conflict of interest."

References

1. Xu, Z., Uncertain Multi-attribute Decision Making. Berlin: Springer, 2015.
2. Zadeh, L. A., "Fuzzy sets", Inf. Control. Vol. 8, No. 3, pp. 338-353, 1965.
3. Atanassov, K. T., "Intuitionistic fuzzy sets", Fuzzy Sets Syst. Vol. 20, No. 2, pp. 87-96, 1986.
4. Smarandache, F., Neutrosophy. Neutrosophic Probability, Set, and Logic. American Research Press: Rehoboth, DE, USA, 1998.
5. Smarandache, F., A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic. American Research Press: Rehoboth, DE, USA, 1999.
6. Wang, H., Smarandache, F., Zhang, Y. et al, "Single valued neutrosophic sets", Multispace Multistruct. Vol. 4, pp. 410-413, 2010.
7. Zhang, C., Li, D., Kang, X., et al, "Neutrosophic fusion of rough set theory: An overview", Comput in Ind. Vol. 115, pp. 103117, 2020.
8. Zhang, C., Li, D., Kang, X., et al, "Multi-attribute group decision making based on multigranulation probabilistic models with interval-valued neutrosophic information", Mathematics. Vol. 8, No. 2, pp.223, 2020.
9. Zhang, C., Li, D., Liang, J., "Multi-granularity three-way decisions with adjustable hesitant fuzzy linguistic multigranulation decision-theoretic rough sets over two universes", Inform Sci. Vol. 507, pp. 665-683, 2020.
10. Rosenfeld, A., Fuzzy graphs. in: Zadeh A., Fu K., Shimura M. (Eds.), Fuzzy Sets and Their Applications, Academic Press, 1975.
11. Mordeson, J., Peng, C., "Operations on fuzzy graphs", Inform Sci. Vol. 79, No. 3-4, pp. 159-170, 1994.
12. Ye, J., "Single-valued neutrosophic minimum spanning tree and its clustering method", J. Intell. Syst. Vol. 23, No. 3, pp. 311-324, 2014.
13. Yang, H., Guo, Z., Liao, X., "On single-valued neutrosophic relations", J. Intell. Fuzzy Syst. Vol. 30, No. 2, pp. 1045-1056, 2016.
14. Dhavaseelan, R., Vikramaprasad, R., Krishnaraj, V., "Certain types of neutrosophic graphs", Int. J. Math. Sci. Appl. Vol. 5, No. 2, pp. 333-339, 2015.

15. Akram, M., Shahzadi, S., “Neutrosophic soft graphs with application”, J. Intell. Fuzzy Syst. Vol. 32, pp. 841-858, 2017.

16. Akram, M., Shahzadi, S., “Operations on single-valued neutrosophic graphs”, J. Uncertain Syst. Vol. 11, No. 1, pp. 1-26, 2017.

17. Broumi, S., Talea, M., Bakali, A., et al. “Single-valued neutrosophic graphs”, J. New Theory. Vol. 10, pp. 86-101, 2016.

18. Shah, N., Hussain, A., “Neutrosophic soft graphs”, Neutrosophic Set Syst. Vol. 11, pp. 31-44, 2016.

19. Akram, M., , M., “Novel applications of single-valued neutrosophic graph structures in decision making”, J. Appl. Math. Comput. Vol.56, pp. 501-532, 2018.

20. Broumi, S., Bakali, A., Talea, M., et al, “Spanning tree problem with neutrosophic edge weights”, Procedia Computer Science. Vol.127, pp. 190-199 (2018).

21. Broumi, S., Dey, A., Talea, M., et al, “Shortest path problem using bellman algorithm under neutrosophic environment”, Complex & Intelligent Systems. Vol.5, No. 4, pp. 409-416 (2019).

22. Karaaslan, F., Davvaz, B., “Properties of single-valued neutrosophic graphs”, Journal of Intelligent and Fuzzy Systems. Vol.7, No. 1, pp. 57-79 (2018).

23. Karaaslan, F., “Gaussian single-valued neutrosophic numbers and its application in multi-attribute decision making”, Neutrosophic Sets and Systems. Vol.22, pp. 101-117 (2018).

24. Garg, H., Nancy, “New logarithmic operational laws and their aplications to multiattribute decision making for single-valued neutrosophic numbers”, J. Cognitive Syst. Vol.52, pp. 931-946 (2018).

25. Smarandache, F., Martin, N., “Plithogenic n-super hypergraph in novel multi-attribute decision making”, International Journal of Neutrosophic Science, Vol.7, No. 1, pp. 8-30, 2020.

26. Martin, N., Smarandache, F., “Plithogenic cognitive maps in decision making”, International Journal of Neutrosophic Science, Vol. 9, No. 1, pp. 9-21, 2020.

27. Singh, P. K., “Plithogenic set for multi-variable data analysis”, International Journal of Neutrosophic Science, Vol. 1, No. 2, pp. 81-89, 2020.

28. Rana, S., Saeed, M., Qayyum, M., et al, “Plithogenic subjective hyper-super-soft matrices with new definitions & local, global, universal subjective ranking model”, International Journal of Neutrosophic Science, Vol. 6, No. 2, pp. 56-79, 2020.

On Finite and Infinite NeutroRings of Type-NR[8,9]

A.A.A. Agboola[◇]
Department of Mathematics, Federal University of Agriculture, Abeokuta, Nigeria.
agboolaaaa@funaab.edu.ng
◇ In commemoration of the 60th birthday of the author

Abstract

NeutroRings are alternatives to the classical rings and they are of different types. NeutroRings in some cases exhibit different algebraic properties, and in some cases they exhibit algebraic properties similar to the classical rings. The objective of this paper is to revisit the concept of NeutroRings and study finite and infinite NeutroRings of type-NR[8,9]. In NeutroRings of type-NR[8,9], the left and right distributive axioms are taking to be either partially true or partially false for some elements; while all other classical laws and axioms are taking to be totally true for all the elements. Several examples and properties of NeutroRings of type-NR[8,9] are presented. NeutroSubrings, NeutroIdeals, NeutroQuotientRings and NeutroRingHomomorphisms of the NeutroRings of type-NR[8,9] are studied with several interesting examples and their basic properties are presented. It is shown that in NeutroRings of type-NR[8,9], the fundamental theorem of homomorphisms of the classical rings holds.
Keywords: NeutroRing, AntiRing, NeutroSubring, NeutroIdeal, NeutroQuotientRing, NeutroRingHomomorphism.

1 Introduction and Preliminaries

In the classical rings $(R, +, .)$, addition and multiplication closure laws are 100% true for all the elements of R. Also, associative and distributive axioms over R are 100% true for all the elements of R. There are no provisions in the classical ring R to have addition and multiplication laws to be either partially true or partially indeterminate or partially false for the elements of R. Also, there are no provisions for associative and distributive axioms over R to be either partially true or partially indeterminate or partially false for the elements of R. Lack of these provisions in the classical rings posses problems because such rings cannot be used to model the real life situations accurately. These problems were addressed by Smarandache in [10] by introducing the concepts of NeutroAlgebraicStructures and AntiAlgebraicStructures. Smarandache further studied these new concepts in [9] and [8] respectively. With these new concepts, a lot of research activities have begun with some papers already published. For instance in [7], Rezaei and Smarandache studied Neutro-BE-algebras and Anti-BE-algebras and they showed that any classical algebra S with n operations (laws and axioms) where $n \geq 1$ will have $(2^n - 1)$ NeutroAlgebras and $(3^n - 2^n)$ AntiAlgebras. In [3], Agboola et al. studied NeutroAlgebras and AntiAlgebras viz-a-viz the classical number systems, in [4], Agboola studied NeutroGroups and in [5], he studied NeutroRings. Also in [2], Agboola revisited NeutroGroups and in [1], he studied AntiGroups. In the present paper, the concept of NeutroRings introduced in [5] is revisited. It is shown that there are 511 types of NeutroRings and 19171 types of AntiRings. In particular, finite and infinite NeutroRings of type-NR[8,9] are studied. In NeutroRings of type-NR[8,9], the left and right distributive axioms are taking to be either partially true or partially false for some elements; while all other classical laws and axioms are taking to be totally true for all the elements. Several examples and properties of NeutroRings of type-NR[8,9] are presented. NeutroSubrings, NeutroIdeals, NeutroQuotientRings and NeutroRingHomomorphisms of the NeutroRings of type-NR[8,9] are studied with several interesting examples and their basic properties are presented. It is shown that in NeutroRings of type-NR[8,9], the fundamental theorem of homomorphisms of the classical rings holds.

Definition 1.1. [8]

(i) A classical operation is an operation well defined for all the set's elements.

Doi :10.5281/zenodo.4135100

Received: Jun 02, 2020 Accepted: Octobre 10, 2020

(ii) A NeutroOperation is an operation partially well defined, partially indeterminate, and partially outer defined on the given set.

(iii) An AntiOperation is an operation that is outer defined for all set's elements.

(iv) A classical law/axiom defined on a nonempty set is a law/axiom that is totally true (i.e. true for all set's elements).

(v) A NeutroLaw/NeutroAxiom (or Neutrosophic Law/Neutrosophic Axiom) defined on a nonempty set is a law/axiom that is true for some set's elements [degree of truth (T)], indeterminate for other set's elements [degree of indeterminacy (I)], or false for the other set's elements [degree of falsehood (F)], where $T, I, F \in [0, 1]$, with $(T, I, F) \neq (1, 0, 0)$ that represents the classical axiom, and $(T, I, F) \neq (0, 0, 1)$ that represents the AntiAxiom.

(vi) An AntiLaw/AntiAxiom defined on a nonempty set is a law/axiom that is false for all set's elements.

(vii) A NeutroAlgebra is an algebra that has at least one NeutroOperation or one NeutroAxiom (axiom that is true for some elements, indeterminate for other elements, and false for other elements).

(viii) An AntiAlgebra is an algebra endowed with at least one AntiOperation or at least one AntiAxiom.

Theorem 1.2. *[7] Let $\mathbb{U}$ be a nonempty finite or infinite universe of discourse and let S be a finite or infinite subset of $\mathbb{U}$. If n classical operations (laws and axioms) are defined on S where $n \geq 1$, then there will be $(2^n - 1)$ NeutroAlgebras and $(3^n - 2^n)$ AntiAlgebras.*

2 NeutroRings Revisited

Definition 2.1. [Classical ring][6]
Let R be a nonempty set and let $+, . : R \times R \to R$ be binary operations of the usual addition and multiplication respectively defined on R. The triple $(R, +, .)$ is called a classical ring if the following conditions $(R1 - R9)$ hold:

(R1) $x + y \in R\ \forall x, y \in R$ [closure law of addition].

(R2) $x + (y + z) = (x + y) + z\ \forall x, y, z \in R$ [axiom of associativity].

(R3) There exists $e \in R$ such that $x + e = e + x = x\ \forall x \in R$ [axiom of existence of neutral element].

(R4) There exists $-x \in R$ such that $x + (-x) = (-x) + x = e\ \forall x \in G$ [axiom of existence of inverse element]

(R5) $x + y = y + x\ \forall x, y \in R$ [axiom of commutativity].

(R6) $x.y \in R\ \forall x, y \in R$ [closure law of multiplication].

(R7) $x.(y.z) = (x.y).z\ \forall x, y, z \in R$ [axiom of associativity].

(R8) $x.(y + z) = (x.y) + (x.z)\ \forall x, y, z \in R$ [axiom of left distributivity].

(R9) $(y + z).x = (y.x) + (z.x)\ \forall x, y, z \in R$ [axiom of right distributivity].

If in addition we have,

(R10) $x.y = y.x\ \forall x, y \in R$ [axiom of commutativity],

then $(R, +, .)$ is called a commutative ring.

Definition 2.2. [NeutroSophication of the laws and axioms of the classical ring]

(NR1) There exist at least three duplets $(x, y), (u, v), (p, q) \in R$ such that $x + y \in R$ (degree of truth T) and $[u + v =$ outer-defined/indeterminate (degree of indeterminacy I) or $p + q \notin R]$ (degree of falsehood F) [NeutroClosure law of addition].

(NR2) There exist at least three triplets $(x, y, z), (p, q, r), (u, v, w) \in R$ such that $x + (y + z) = (x + y) + z$ (degree of truth T) and $[[p + (q + r)]$or$[(p + q) + r] =$ outer-defined/indeterminate (degree of indeterminacy I) or $u + (v + w) \neq (u + v) + w]$ (degree of falsehood F) [NeutroAxiom of associativity (NeutroAssociativity)].

Doi :10.5281/zenodo.4135100

(NR3) There exists an element $e \in R$ such that $x+e = x+e = x$ and $[[x+e]$or$[e+x] =$ outer-defined/indeterminate or $x + e \neq x \neq e + x]$ for at least one $x \in R$ [NeutroAxiom of existence of neutral element (NeutroNeutralElement)].

(NR4) There exists $-x \in R$ such that $x+(-x) = (-x)+x = e$ and $[[-x+x]$or$[x+(-x)] =$ outer-defined/indeterminate or $-x+x \neq e \neq x+(-x)]$ for at least one $x \in R$ [NeutroAxiom of existence of inverse element (NeutroInverseElement)].

(NR5) There exist at least three duplets $(x, y), (u, v), (p, q) \in R$ such that $x+y = y+x$ and $[[p+q]$or$[q+p] =$ outer-defined/indeterminate (degree of indeterminacy I) or $u + v \neq v + u]$ (degree of falsehood F) [NeutroAxiom of commutativity (NeutroCommutativity)].

(NR6) There exist at least three duplets $(x, y), (p, q), (u, v) \in R$ such that $x.y \in R$ (degree of truth T) and $[u.v =$ outer-defined/indeterminate (degree of indeterminacy I) or $p.q \notin R]$ (degree of falsehood F) NeutroClosure law of multiplication].

(NR7) There exist at least three triplets $(x, y, z), (p, q, r), (u, v, w) \in R$ such that $x.(y.z) = (x.y).z$ (degree of truth T) and $[[p.(q.r)]$or$[(p.q).r] =$ outer-defined/indeterminate (degree of indeterminacy I) or $u.(v.w) \neq (u.v).w]$ (degree of falsehood F) [NeutroAxiom of associativity (NeutroAssociativity)].

(NR8) There exist at least three triplets $(x, y, z), (p, q, r), (u, v, w) \in R$ such that $x.(y + z) = (x.y) + (x.z)$ (degree of truth T) and $[[p.(q + r)]$or$[(p.q) + (p.r)] =$ outer-defined/indeterminate (degree of indeterminacy I) or $u.(v + w) \neq (u.v) + (u.w)]$ (degree of falsehood F) [NeutroAxiom of left distributivity (NeutroLeftDistributivity)].

(NR9) There exist at least three triplets $(x, y, z), (p, q, r), (u, v, w) \in R$ such that $(y + z).x = (y.x) + (z.x)$ (degree of truth T) and $[[(v + w).u]$or$[(v.u) + (w.u)] =$ outer-defined/indeterminate (degree of indeterminacy I) or $(v + w).u \neq (v.u) + (w.u)]$ (degree of falsehood F) [NeutroAxiom of right distributivity (NeutroRightDistributivity)].

(NR10) There exist at least three duplets $(x, y), (p, q), (u, v) \in R$ such that $x.y = y.x$ (degree of truth T) and $[[p.q]$or$[q.p] =$ outer-defined/indeterminate (degree of indeterminacy I) or $u.v \neq v.u]$ (degree of falsehood F) [NeutroAxiom of commutativity (NeutroCommutativity)].

Definition 2.3. [AntiSophication of the law and axioms of the classical ring]

(AR1) For all the duplets $(x, y) \in R$, $x + y \notin R$ [AntiClosure law of addition].

(AR2) For all the triplets $(x, y, z) \in R$, $x + (y + z) \neq (x + y) + z$ [AntiAxiom of associativity (AntiAssociativity)].

(AR3) There doest not exist an element $e \in R$ such that $x + e = x + e = x\ \forall x \in R$ [AntiAxiom of existence of neutral element (AntiNeutralElement)].

(AR4) There does not exist $-x \in R$ such that $x + (-x) = (-x) + x = e\ \forall x \in R$ [AntiAxiom of existence of inverse element (AntiInverseElement)].

(AR5) For all the duplets $(x, y) \in R$, $x + y \neq y + x$ [AntiAxiom of commutativity (AntiCommutativity)].

(AR6) For all the duplets $(x, y) \in R$, $x.y \notin R$ [AntiClosure law of multiplication].

(AR7) For all the triplets $(x, y, z) \in R$, $x.(y.z) \neq (x.y).z$ [AntiAxiom of associativity (AntiAssociativity)].

(AR8) For all the triplets $(x, y, z) \in R$, $x.(y + z) \neq (x.y) + (x.z)$ [AntiAxiom of left distributivity (AntiLeftDistributivity)].

(AR9) For all the triplets $(x, y, z) \in R$, $(y + z).x \neq (y.x) + (z.x)$ [AntiAxiom of right distributivity (AntiRightDistributivity)].

(AR10) For all the duplets $(x, y) \in R$, $x.y \neq y.x$ [AntiAxiom of commutativity (AntiCommutativity)].

Definition 2.4. [NeutroRing]
A NeutroRing NR is an alternative to the classical ring R that has at least one NeutroLaw or at least one of $\{NR1, NR2, NR3, NR4, NR5, NR6, NR7, NR8, NR9\}$ with no AntiLaw or AntiAxiom.

Definition 2.5. [AntiRing]
An AntiRing AR is an alternative to the classical ring R that has at least one AntiLaw or at least one of $\{AR1, AR2, AR3, AR4, AR5, AR6, AR7, AR8, AR9\}$.

Definition 2.6. [NeutroCommutativeRing]
A NeutroNoncommutativeRing NR is an alternative to the classical noncommutative ring R that has at least one NeutroLaw or at least one of $\{NR1, NR2, NR3, NR4, NR5, NR6, NR7, NR8, NR9\}$ and $NR10$ with no AntiLaw or AntiAxiom.

Definition 2.7. [AntiCommutativeRing]
An AntiCommutativeRing AR is an alternative to the classical commutative ring R that has at least one AntiLaw or at least one of $\{AR1, AR2, AR3, AR4, AR5, AR6, AR7, AR8, AR9\}$ and $AR10$.

Proposition 2.8. *Let $(R, +, .)$ be a finite or infinite classical ring. Then:*

(i) *There are 511 types of NeutroRings.*

(ii) *There are 19171 types of AntiRings.*

Proof. Follows from Theorem 1.2. □

Proposition 2.9. *Let $(R, +, .)$ be a finite or infinite classical commutative ring. Then:*

(i) *There are 1023 types of NeutroCommutativeRings.*

(ii) *There are 58025 types of AntiCommutativeRings.*

Proof. Follows from Theorem 1.2. □

Remark 2.10. It is evident from Proposition 2.8 and Proposition 2.9 that there are many types of NeutroRings and NeutroCommutativeRings. The type of NeutroRings studied by Agboola in [5] are those for which $NR1-NR10$ are all true.

Example 2.11. (i) Let $NR = \mathbb{Z}$ and let $\oplus$ be a binary operation of ordinary addition and for all $x, y \in NR$, let $\odot$ be a binary operation defined on NR as $x \odot y = \sqrt{xy}$. Then $(NR, \oplus, \odot)$ is a NeutroRing.

(ii) Let $NR = \mathbb{Q}$ and let $\oplus$ be a binary operation of ordinary addition and for all $x, y \in NR$, let $\odot$ be a binary operation defined on NR as $x \odot y = x/y$. Then $(NR, \oplus, \odot)$ is a NeutroRing.

(iii) Let $AR = \mathbb{N}$ and let $\ominus$ and $\otimes$ be two binary operations of ordinary subtraction and ordinary multiplication respectively defined on AR. Then $(AR, \ominus, \otimes)$ is an AntiRing.

(iv) Let $AR = \mathbb{N}$ and let $\oplus$ and $\otimes$ be two binary operations of ordinary addition and ordinary multiplication respectively defined on AR. Then $(AR, \oplus, \otimes)$ is an AntiRing.

Definition 2.12. Let $(NR, +, .)$ be a NeutroRing.

(i) NR is called a finite NeutroRing of order n if the cardinality of NR is n that is $o(NR) = n$. Otherwise, NR is called an infinite NeutroRing and we write $o(NR) = \infty$.

(ii) NR is called a NeutroRing with unity if there exists a multiplicative unit element $u \in NR$ such that $ux = xu = x$ for at least one $x \in R$.

(iii) If there exists a least positive integer n such that $nx = e$ for at least one $x \in NR$, then NR is called a NeutroRing of characteristic n. If no such n exists, then NR is called a NeutroRing of characteristic zero.

(iv) An element $x \in NR$ is called an idempotent element if $x^2 = x$.

(v) An element $x \in NR$ is called a nilpotent element if for the least positive integer n, we have $x^n = e$.

(vi) An element $e \neq x \in NR$ is called a zero divisor element if there exists an element $e \neq y \in NR$ such that $xy = e$ or $yx = e$.

(vii) An element $x \in NR$ is called a multiplicative inverse element if there exists at least one $y \in NR$ such that $xy = yx = u$ where u is the multiplicative unity element in NR.

Definition 2.13. Let $(NR, +, .)$ be a NeutroCommutativeRing with unity. Then

(i) NR is called a NeutroIntegralDomain if NR has no at least one zero divisor element.

(ii) NR is called a NeutroField if NR has at least one multiplicative inverse element.

Example 2.14. Let $NR = \mathbb{Z}_5 = \{0, 1, 2, 3, 4\}$ and let $\oplus$ and $\odot$ be two binary operations defined on NR by

$$x \oplus y = x + y - 1, \;\; x \odot y = x + xy \;\; \forall\, x, y \in NR.$$

It is clear that $(NR, \oplus)$ is an abelian group.

(1) **NeutroAssociativity:** Let $x, y, z \in NR$. Then

$$\begin{array}{rcl} x \odot (y \odot z) & = & x + xy + xyz, \\ (x \odot y) \odot z & = & x + xy + xz + xyz, \\ \therefore \quad x + xy + xyz & = & x + xy + xz + xyz \\ \Rightarrow \quad xz & = & 0 \\ \therefore \quad x & = & 0 \;\text{ or }\; z = 0. \end{array}$$

This shows that only the triplets $(0, y, z), (x, y, 0), (0, y, 0)$ can verify associativity with 60% degree of associativity.

(2) **NeutroLeftDistributivity:** Let $x, y, z \in NR$. Then

$$\begin{array}{rcl} x \odot (y \oplus z) & = & x + xy + xz - x, \\ (x \odot y) \oplus (x \odot z) & = & x + xy + x + xz - 1, \\ \therefore \quad x + xy + xz - x & = & x + xy + x + xz - 1 \\ \Rightarrow \quad 2x & = & 1 \\ \therefore \quad x & = & 3. \end{array}$$

This shows that only the triplet $(3, y, z)$ can verify left distributivity with 20% degree of left distributivity.

(3) **NeutroRightDistributivity:** Let $x, y, z \in NR$. Then

$$\begin{array}{rcl} (y \oplus z) \odot x & = & y + z - 1 + yx + zx - x, \\ (y \odot x) \oplus (z \odot x) & = & y + yx + z + zx - 1, \\ \therefore \quad y + z - 1 + yx + zx - x & = & y + yx + z + zx - 1 \\ \Rightarrow \quad -x & = & 0 \\ \therefore \quad x & = & 0. \end{array}$$

This shows that only the triplet $(0, y, z)$ can verify right distributivity with 20% degree of right distributivity.

(4) **NeutroCommutativity:** Let $x, y \in NR$. Then

$$\begin{array}{rcl} x \odot y & = & x + xy, \\ y \odot x & = & y + yx, \\ \therefore \quad x + xy & = & y + yx \\ \Rightarrow \quad x & = & y \\ \therefore \quad x & = & y. \end{array}$$

This shows that only the duplet (x, x) can verify commutativity with 20% degree of commutativity.

We have just shown according to Definition 2.6 that $(NR, \oplus, \odot)$ is a NeutroRing.

Example 2.15. Let $NR = \{a, b, c, d\}$ and let $''+''$ and $''.''$ be binary operations defined on NR as shown in the Cayley tables below:

$+$	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

$.$	a	b	c	d
a	a	b	c	d
b	a	c	b	c
c	c	d	c	d
d	d	a	d	a

It is clear that $(NR, +)$ is an abelian group. From the tables we have:

(1) **NeutroAssociativity:**

$$\begin{aligned} a(bc) &= (ab)c = b, \\ b(bb) &= b \text{ but } (bb)b = d \neq b. \end{aligned}$$

This shows NeutroAssociativity of $''+''$.

(2) **NeutroLeftDistributivity:**

$$\begin{aligned} a(b+c) &= ab + ac = d, \\ b(c+d) &= c \text{ but } bc + bd = d \neq c. \end{aligned}$$

This shows NeutroLeftDistributivity of $''.''$ over $''+''$.

(3) **NeutroRightDistributivity:**

$$\begin{aligned} (b+c)c &= bc + cc = d, \\ (b+c)a &= d \text{ but } ba + ca = c \neq d. \end{aligned}$$

This shows NeutroRightDistributivity of $''.''$ over $''+''$.

(4) **NeutroCommutativity:**

$$\begin{aligned} ac &= ca = a, \\ bc &= b \text{ but } cb = d \neq b. \end{aligned}$$

This shows NeutroCommutativity of $''.''$.

We have just shown according to Definition 2.6 that $(NR, +, .)$ is a NeutroRing.

Example 2.16. From Example 2.15, we note that $e = a$ is the additive neutral element. We now have the following:

(i) NR is a NeutroCommutativeRing with unity since $aa = a, ac = ca = c, ad = da = d$.

(ii) $\{a, c\}$ are idempotent elements.

(iii) $\{d\}$ is a nilpotent element.

(iv) $\{b, d\}$ are zero divisor elements.

(v) $\{a, d\}$ are invertible elements.

(vi) NR is not a NeutroIntegralDomain.

(vii) NR is a NeutroField.

(viii) NR is a NeutroCommutativeRing of characteristic 2.

Example 2.17. Let $\mathbb{U} = \{e, a, b, c, d, f\}$ be a universe of discourse and let $NR = \{e, a, b, c\}$. Suppose that $*$ and $\circ$ are two binary operations defined on NR as shown in the Cayley tables below:

$\circ$	e	a	b	c
e	e	a	b	c
a	a	b or e	c	b
b	b	c	c or e	a
c	c	b	a	f

$*$	e	a	b	c
e	e	b	c	a or b or e
a	a	c	e	d
b	b	e	a	c
c	c	a	b	e

It is clear that $(NR, \circ)$ is a NeutroGroup. Now consider the following:

(i) **NeutroAssociativity of $*$:** $a*(b*b) = (a*b)*b = c$, $b*(a*b) = b$ but $(b*a)*b = c \neq b$, $a*(b*c) = d$ (outer-defined), $(a*b)*c = e*c =$ indeterminate.

(ii) **NeutroLeftDistributivity of $*$ over $\circ$:** $e*(e\circ e) = (e*e)\circ(e*e) = e$, $a*(b\circ e) = e$ but $(a*b)\circ(a*e) = a \neq e$, $a*(b\circ c) = e$ but $(a*b)\circ(a*c) = e\circ d = ?$.

(iii) **NeutroRightDistributivity of $*$ over $\circ$:** $(e\circ e)*e) = (e*e)\circ(e*e) = e$, $(b\circ c)*a = c$ but $(b*a)\circ(c*a) = a \neq c$, $(e\circ e)*c = e*e = ?$ and $(e*c)\circ(e*c) = ?$.

(iv) **NeutroCummutativity of $*$:** $e*e = c*c = e$, $b*c = c$ but $c*b = b \neq c$, $e*c =$ indeterminate but $c*e = c$.

Hence $(NR, \circ, *)$ is a NeutroRing.

3 Finite and Infinite NeutroRings of Type-NR[8,9]

In this section, we are going to study a type of NeutroRings $(NR, \circ, *)$ where $R1, R2, R3, R4, R5, R6, R7, R10$ are totally true for all the elements of NR, and where $R8$ and $R9$ are either partially true or partially false for some elements of NR. This type of NeutroRings will be called NeutroRings of type-NR[8,9].

Example 3.1. Let $NR = \mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$ and let $\circ$ and $*$ be two binary operations defined on NR by

$$x\circ y = x + y, \quad x*y = x + y + xy \quad \forall x, y \in NR$$

where $''+''$ is addition modulo 6. Then $(NR, \circ, *)$ is a finite NeutroRing of type-NR[8,9]. To see this, consider the Cayley tables below.

$\circ$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

$*$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	3	5	1	3	5
2	2	5	2	5	2	5
3	3	1	5	3	1	5
4	4	3	2	1	0	5
5	5	5	5	5	5	5

It is clear from the tables that $(NR, \circ)$ is an abelian group with $e = 0$ as the identity element, and that $(NR, *)$ is a commutative semigroup. It remains to show that the two distributive axioms are NeutroAxioms.

(i) **NeutroLeftDistributivity of $*$ over $\circ$:** Let $x, y, z \in NR$. Then $x*(y\circ z) = x + y + z + xy + xz$ and $(x*y)\circ(x*z) = 2x + y + z + xy + xz$. For left distributivity to hold, we must have $2x = x$ from which we obtain $x = 0$. Hence, only the triplets $(0, y, z), (0, y, 0), (0, 0, z), (0, 0, 0)$ can verify the left distributivity of $*$ over $\circ$ with 66.67% degree of distributivity. Hence, $*$ is NeutroLeftDistributive over $\circ$ in NR.

(ii) **NeutroRightDistributivity of $*$ over $\circ$:** It can similarly be shown that $*$ is NeutroRightDistributive over $\circ$ with 66.67% degree of distributivity.

Hence, $(NR, \circ, *)$ is a finite NeutroRing of type-NR[8,9].

Example 3.2. Let $NR = \mathbb{Z}$ or $\mathbb{Q}$ or $\mathbb{R}$ or $\mathbb{C}$ and let $\circ$ and $*$ be two binary operations defined on NR by

$$x\circ y = x + y, \quad x*y = x + y + xy \quad \forall x, y \in NR$$

where $''+''$ is the ordinary addition of integers or rationals or reals or complex numbers. It is clear that $(NR, \circ)$ is an abelian group with $e = 0$ as the identity element, and that $(NR, *)$ is a commutative semigroup. It remains to show that the two distributive axioms are NeutroAxioms.

(i) **NeutroLeftDistributivity of $*$ over $\circ$:** Let $x, y, z \in NR$. Then $x*(y\circ z) = x + y + z + xy + xz$ and $(x*y)\circ(x*z) = 2x + y + z + xy + xz$. For left distributivity to hold, we must have $2x = x$ from which we obtain $x = 0$. Hence, only the triplets $(0, y, z), (0, y, 0), (0, 0, z), (0, 0, 0)$ can verify the left distributivity of $*$ over $\circ$. Hence, $*$ is NeutroLeftDistributive over $\circ$ in NR.

(ii) **NeutroRightDistributivity of $*$ over $\circ$:** It can similarly be shown that that $*$ is NeutroRightDistributive over $\circ$.

Hence, $(NR, \circ, *)$ is an infinite NeutroRing of type-NR[8,9].

Proposition 3.3. *Let $(NR_i, \circ, *), i = 1, 2$ be NeutroRings of type-NR[8,9]. In the Cartesian product $NR_1 \times NR_2$ of NR_i, let $\oplus$ and $\odot$ be two binary operations defined $\forall (w, x), (y, z) \in NR_1 \times NR_2$ as follows:*

$$\begin{aligned} (w, x) \oplus (y, z) &= (w \circ y, x \circ z) \\ (w, x) \odot (y, z) &= (w * y, x * z). \end{aligned}$$

Then $(NR_1 \times NR_2, \oplus, \odot)$ is a NeutroRing of type-NR[8,9].

Proof. Follows from Definition 2.2. □

Proposition 3.4. *Let $(NR, +, .)$ be a NeutroRing of type-NR[8,9] and let e be the identity element in NR with respect to $''+''$. Then for some $x, y, z \in NR$, we have:*

(i) $x.e \neq e$.

(ii) $x.(-y) \neq -(x.y) \neq (-x).y$.

(iii) $x.(y - z) \neq x.y - x.z$.

(iv) $(y - z).x \neq y.x - z.x$.

Proof. Since $''.''$ is NeutroDistributive over $''+''$, the required results follow. □

Proposition 3.5. *Let $(NR, +, .)$ be a NeutroRing of type-NR[8,9]. Then for some $w, x, y, z \in NR$, we have:*

(i) $(w + x).(y + z) \neq (w.y + w.z) + (x.y + x.z)$.

(ii) $(w + x).(y - z) \neq (w.y + x.y) - (w.z + x.z)$.

(iii) $(w - x).(y - z) \neq (w.y + x.z) - (w.z + x.y)$.

(iv) $(w + x).(w - x) \neq (w.w - x.x) + (x.w - w.x)$.

Proof. Since $''.''$ is NeutroDistributive over $''+''$, the required results follow. □

Proposition 3.6. *Let $(NR, +, .)$ be a NeutroRing of type-NR[8,9] and let $m, n \in \mathbb{N}$. Then $\forall x \in NR$, we have:*

(i) $x^m.x^n = x^{m+n}$.

(ii) $(x^m)^n = (x^n)^m = x^{mn}$.

Proof. Since $''.''$ is associative, the required results follow. □

Definition 3.7. Let $(NR, \circ, *)$ be a NeutroRing of type-NR[8,9] and let NS be nonempty subset of NR.

(i) NS is called a NeutroSubring of NR if $(NS, \circ, *)$ is also a NeutroRing of type-NR[8,9].

(ii) NS is called a QuasiNeutroSubring of NR if $(NS, \circ, *)$ is a NeutroRing of the type different from the type of the parent NeutroRing NR.

The only trivial NeutroSubring of NR is NR.

Proposition 3.8. *There exist NeutroRings of type-NR[8,9] with only trivial NeutroSubrings.*

Proof. Consider the structure $(NR, \circ, *)$ such that $NR = \mathbb{Z}_6$ and $\forall\ x, y \in NR$, we have $x \circ y = x + y + 1, x * y = x + y + 3xy$ and consider the structure $(NS, \circ, *)$ where $NS = \mathbb{Z}$ and $\forall\ x, y \in NS$, $x \circ y = x + y - 7, x * y = x + y - 3xy$. It can be shown that NR and NS are NeutroRings of type-NR[8,9] with only trivial NeutroSubrings. □

Example 3.9. Let $(NR, \circ, *)$ be the NeutroRing of Example 3.1 and let $NS_1 = \{0, 3\}$ and $NS_2 = \{0, 2, 4\}$ be two subsets of NR. It can easily be shown that $(NS_1, \circ, *)$ and $(NS_2, \circ, *)$ are NeutroRings of type-NR[8,9] and consequently they are NeutroSubrings of NR. It is observed that $NS_1 \cap NS_2 = \{0\}$ and $NS_1 \cup NS_2 = \{0, 2, 3, 4\}$ are not NeutroSubrings of NR. Also, $NS_1 \times NS_2 = \{(0, 0), (0, 2), (0, 4), (3, 0), (3, 3), (3, 4)\}$ is a NeutroSubring of $NR \times NR$.

Example 3.10. Let $(NR, \circ, *)$ be the NeutroRing of Example 3.2 and let $NS_1 = 2\mathbb{Z}$, $NS_2 = 3\mathbb{Z}$ and $NS_3 = 4\mathbb{Z}$ be three subsets of NR. It can easily be shown that NS_1, NS_2 and NS_3 are NeutroSubrings of NR. Generally for positive integers $n \geq 2$, it can be shown that $NS = n\mathbb{Z}$ are NeutroSubrings of NR. It is observed that $NS_1 \cap NS_2 = 6\mathbb{Z}$, $NS_1 \cap NS_3 = 4\mathbb{Z}$, $NS_2 \cap NS_3 = 12\mathbb{Z}$ and $NS_1 \cup NS_3 = 2\mathbb{Z}$ are NeutroSubrings of NR. However, $NS_1 \cup NS_2$ and $NS_2 \cup NS_3$ are not NeutroSubrings of NR.

Proposition 3.11. *Let $(NR, \circ, *)$ be a NeutroRing of type-NR[8,9] and let $\{NS_i\}, i = 1, 2$ be NeutroSubrings of NR. Then*

(i) $NS = NS_1 \cap NS_2$ is not necessarily a NeutroSubring of NR.

(ii) $NS = NS_1 \times NS_2$ is a NeutroSubring of $NR \times NR$.

(iii) $NS = NS_1 \cup NS_2$ is not necessarily a NeutroSubring of NR.

Definition 3.12. Let $(NR, \circ, *)$ be a NeutroRing of type-NR[8,9]. A nonempty subset NI of NR is called a NeutroIdeal of NR if the following conditions hold:

(i) NI is a NeutroSubring of NR.

(ii) $x \in NI$ and $r \in NR$ imply that at least one $r * x$ or $x * r \in NI$ for all $r \in NR$.

Definition 3.13. Let $(NR, \circ, *.)$ be a NeutroRing of type-NR[8,9]. A nonempty subset NI of NR is called a QuasiNeutroIdeal of NR if the following conditions hold:

(i) NI is a QuasiNeutroSubring of NR.

(ii) $x \in NI$ and $r \in NR$ imply that at least one $x * r$ or $r * x \in NI$ for all $r \in NR$.

Example 3.14. Let $NI_1 = NS_1 = \{0, 3\}$ and $NI_2 = NS_2 = \{0, 2, 4\}$ be NeutroSubrings of Example 3.9. Then for NI_1, we have $0 * 0 = 0, 1 * 0 = 1, 2 * 0 = 2, 3 * 0 = 3, 4 * 0 = 4, 5 * 0 = 5$ and $0 * 3 = 3, 1 * 3 = 1, 2 * 3 = 5, 3 * 3 = 3, 4 * 3 = 1, 5 * 3 = 5$. Accordingly, NI_1 is a NeutroIdeal.

Also for NI_2, we have $0 * 0 = 0, 1 * 0 = 1, 2 * 0 = 2, 3 * 0 = 3, 4 * 0 = 4, 5 * 0 = 5, 0 * 2 = 2, 1 * 2 = 5, 2 * 2 = 2, 3 * 2 = 5, 4 * 2 = 2, 5 * 2 = 5$ and $0 * 4 = 4, 1 * 4 = 3, 2 * 4 = 2, 3 * 4 = 1, 4 * 4 = 0, 5 * 4 = 5$. Accordingly, NI_2 is a NeutroIdeal.

Example 3.15. Let $NI_1 = NS_1 = 2\mathbb{Z}$, $NI_2 = NS_2 = 3\mathbb{Z}$ and $NI_3 = NS_3 = 4\mathbb{Z}$ be NeutroSubrings of Example 3.10. It can easily be shown that NI_1, NI_2 and NI_3 are NeutroIdeals. Generally, $NI = n\mathbb{Z}$ are NeutroIdeals for $n \geq 2$.

Definition 3.16. Let $(NR, \circ, *)$ be a NeutroRing of type-NR[8,9] and let NI be a NeutroIdeal of NR. The set NR/NI is defined by

$$NR/NI = \{x \circ NI : x \in NR\}.$$

For $x \circ NI, y \circ NI \in NR/NI$ with $x, y \in NR$, let $\oplus$ and $\odot$ be binary operations on NR/NI defined as follows:

$$\begin{aligned} (x \circ NI) \oplus (y \circ NI) &= (x \circ y) \circ NI, \\ (x \circ NI) \odot (y \circ NI) &= (x * y) \circ NI. \end{aligned}$$

If the triple $(NR/NI, \oplus, \odot)$ is a NeutroRing of type-NR[8,9], it will be called a NeutroQuotientRing.

Example 3.17. Let $NI_1 = \{0, 3\}$ and $NI_2 = \{0, 2, 4\}$ be NeutroIdeals of Example 3.14. For NI_1, we have

$$NR/NI_1 = \{NI_1, 1 + NI_1, 2 + NI_1\}$$

and the compositions of elements of NR/NI_1 according to Definition 3.16 are given in the Cayley tables:

$\oplus$	NI_1	$1+NI_1$	$2+NI_1$
NI_1	NI_1	$1+NI_1$	$2+NI_1$
$1+NI_1$	$1+NI_1$	$2+NI_1$	NI_1
$2+NI_1$	$2+NI_1$	NI_1	$1+NI_1$

$\odot$	NI_1	$1+NI_1$	$2+NI_1$
NI_1	NI_1	$1+NI_1$	$2+NI_1$
$1+NI_1$	$1+NI_1$	NI_1	$2+NI_1$
$2+NI_1$	$2+NI_1$	$2+NI_1$	$2+NI_1$

It can easily be deduced from the Cayley tables that $(NR/NI_1, \oplus, \odot)$ is a NeutroRing of type-NR[8,9] with $e = NI_1$ as the identity element.

For NI_2, we have

$$NR/NI_2 = \{NI_2, 1 + NI_2\}$$

and the compositions of elements of NR/NI_2 according to Definition 3.16 are given in the Cayley tables:

$\oplus$	NI_2	$1+NI_2$
NI_2	NI_2	$1+NI_2$
$1+NI_2$	$1+NI_2$	NI_2

$\odot$	NI_2	$1+NI_2$
NI_2	NI_2	$1+NI_2$
$1+NI_2$	$1+NI_2$	$1+NI_2$

It can easily be deduced from the Cayley tables that $(NR/NI_2, \oplus, \odot)$ is a NeutroRing of type-NR[8,9] with $e = NI_2$ as the identity element.

Example 3.18. Let $NI_1 = 2\mathbb{Z}$, $NI_2 = 3\mathbb{Z}$ and $NI_3 = 4\mathbb{Z}$ be NeutroIdeals of Example 3.15. For NI_1, we have

$$NR/NI_1 = \{NI_1, 1 + NI_1\}$$

and the compositions of elements of NR/NI_1 according to Definition 3.16 are given in the Cayley tables:

$\oplus$	NI_1	$1+NI_1$
NI_1	NI_1	$1+NI_1$
$1+NI_1$	$1+NI_1$	NI_1

$\odot$	NI_1	$1+NI_1$
NI_1	NI_1	$1+NI_1$
$1+NI_1$	$1+NI_1$	$1+NI_1$

It can easily be deduced from the Cayley tables that $(NR/NI_1, \oplus, \odot)$ is a NeutroRing of type-NR[8,9] with $e = NI_1$ as the identity element.

For NI_2, we have

$$NR/NI_2 = \{NI_2, 1 + NI_2, 2 + NI_2\}$$

and the compositions of elements of NR/NI_2 according to Definition 3.16 are given in the Cayley tables:

$\oplus$	NI_2	$1+NI_2$	$2+NI_2$
NI_2	NI_2	$1+NI_2$	$2+NI_2$
$1+NI_2$	$1+NI_2$	$2+NI_2$	NI_2
$2+NI_2$	$2+NI_2$	NI_2	$1+NI_2$

$\odot$	NI_2	$1+NI_2$	$2+NI_2$
NI_2	NI_2	$1+NI_2$	$2+NI_2$
$1+NI_2$	$1+NI_2$	NI_2	$2+NI_2$
$2+NI_2$	$2+NI_2$	$2+NI_2$	$2+NI_2$

It can easily be deduced from the Cayley tables that $(NR/NI_2, \oplus, \odot)$ is a NeutroRing of type-NR[8,9] with $e = NI_2$ as the identity element.

For NI_3, we have

$$NR/NI_3 = \{NI_3, 1 + NI_3, 2 + NI_3, 3 + NI_3\}$$

and the compositions of elements of NR/NI_3 according to Definition 3.16 are given in the Cayley tables:

$\oplus$	NI_3	$1+NI_3$	$2+NI_3$	$3+NI_3$
NI_3	NI_3	$1+NI_3$	$2+NI_3$	$3+NI_3$
$1+NI_3$	$1+NI_3$	$3+NI_3$	$1+NI_3$	$3+NI_3$
$2+NI_3$	$2+NI_3$	$1+NI_3$	NI_3	$3+NI_3$
$3+NI_3$	$3+NI_3$	$3+NI_3$	$3+NI_3$	$3+NI_3$

$\odot$	NI_3	$1+NI_3$	$2+NI_3$	$3+NI_3$
NI_3	NI_3	$1+NI_3$	$2+NI_3$	$3+NI_3$
$1+NI_3$	$1+NI_3$	$2+NI_3$	$3+NI_3$	NI_3
$2+NI_3$	$2+NI_3$	$3+NI_3$	NI_3	$1+NI_3$
$3+NI_3$	$3+NI_3$	NI_3	$1+NI_3$	$2+NI_3$

It can easily be deduced from the Cayley tables that $(NR/NI_3, \oplus, \odot)$ is a NeutroRing of type-NR[8,9] with $e = NI_3$ as the identity element.

Proposition 3.19. *Let $(NR, +, .)$ be a NeutroRing of type-NR[8,9] and let NI be a NeutroIdeal of NR. For $x+NI, y+NI \in NR/NI$ with $x, y \in NR$, let $\oplus$ and $\odot$ be binary operations on NR/NI defined as follows:*

$$\begin{aligned}(x + NI) \oplus (y + NI) &= (x + y) + NI,\\ (x + NI) \odot (y + NI) &= (xy) + NI.\end{aligned}$$

Then the triple $(NR/NI, \oplus, \odot)$ is a NeutroRing of type-NR[8,9] with $e = NI$ as the identity element.

Proof. Suppose that $(NR, +, .)$ is a NeutroRing of type-NR[8,9] and suppose that NI is NeutroIdeal of NR. That the binary operations $\oplus$ and $\odot$ on NR/NI are well-defined are the same as for the classical rings. It is clear that $(NR/NI, \oplus)$ is an abelian group with $e = NI$ as the identity element and that $(NR/NI, \odot)$ is a commutative semigroup. Since NR is of type-NR[8,9], it follows that there exists at least a triplet $(x, y, z) \in NR$ such that $x(y + z) \neq xy + xz$ and $(y + z)x \neq yx + zx$. Consequently,

$$\begin{aligned}(x + NI) \odot ((y + NI) \oplus (z + NI)) &= x(y + z) + NI\\ &\neq (xy + xz) + NI\\ &= [(x + NI) \odot (y + NI)] \oplus [(x + NI) \odot (z + NI)] \text{ and}\\ ((y + NI) \oplus (z + NI)) \odot (x + NI) &= (y + z)x + NI\\ &\neq (yx + zx) + NI\\ &= [(y + NI) \odot (x + NI)] \oplus [(z + NI) \odot (x + NI)].\end{aligned}$$

Hence, $(NR/NI, \oplus, \odot)$ is a NeutroRing of type-NR[8,9] with $e = NI$ as the identity element. □

Definition 3.20. Let $(NR, +, .)$ and $(NS, +', .')$ be any two NeutroRings of type-NR[8,9]. The mapping $\phi : NR \to NS$ is called a NeutroRingHomomorphism if ϕ preserves the binary operations of NR and NS that is if for at least a duplet $(x, y) \in NR$, we have:

$$\begin{aligned}\phi(x + y) &= \phi(x) +' \phi(y),\\ \phi(x.y) &= \phi(x).'\phi(y).\end{aligned}$$

The kernel of ϕ denoted by $Ker\phi$ is defined as

$$Ker\phi = \{x : \phi(x) = e_{NR}\}.$$

The image of ϕ denoted by $Im\phi$ is defined as

$$Im\phi = \{y \in NS : y = \phi(x) \text{ for at least one } y \in NS\}.$$

If in addition ϕ is a NeutroBijection, then ϕ is called a NeutroRingIsomorphism and we write $NR \cong NS$. NeutroRingEpimorphism, NeutroRingMonomorphism, NeutroRingEndomorphism and NeutroRingAutomorphism are defined similarly.

Example 3.21. Let $(NR, +, *)$ be the NeutroRing of Example 3.1.

(i) Let $\phi : NR \to NR$ be a mapping defined by

$$\phi(x) = 2 * x \quad \forall x \in NR.$$

Then, ϕ is not a NeutroRingHomomorphism. Since $*$ is NeutroDistributive over $''+''$, we have for $x, y \in NR$,

$$\begin{aligned}\phi(x + y) &= 2 * (x + y)\\ &\neq 2 * x + 2 * y\\ &= \phi(x) + \phi(y).\end{aligned}$$

This shows that ϕ does not preserve $''+''$. However since $*$ is associative and $2 * 2 = 2$, we have $\forall x, y \in NR$

$$\begin{aligned}\phi(x * y) &= 2 * (x * y)\\ &= (2 * x) * (2 * y)\\ &= \phi(x) * \phi(y).\end{aligned}$$

This shows that ϕ preserves $*$. Accordingly, ϕ is not a NeutroRingHomomorphism.

(ii) Let $\phi : NR \times NR \to NR$ be a projection defined by

$$\phi(x, y) = x \quad \forall x, y \in NR.$$

It can easily be shown that ϕ is a NeutroRingHomomorphism with

$$\begin{array}{rcl} Ker\phi & = & \{(0,0),(0,1),(0,2),(0,3),(0,4),(0,5)\} \quad \text{and} \\ Im\phi & = & \{0,1,2,3,4,5\} = NR. \end{array}$$

It can be shown that $Ker\phi$ is a NeutroIdeal of $NR \times NR$.

Example 3.22. Let $NR/NI_1 = \{NI_1, 1 + NI_1, 2 + NI_1\}$ be the NeutroQuotientRing of Example 3.17 and let $\phi : NR \to NR/NI_1$ be a mapping defined by $\phi(x) = x + NI_1$ $\forall x \in NR$. Then

$$\begin{array}{rcl} \phi(0) & = & \phi(3) = NI_1, \\ \phi(1) & = & \phi(4) = 1 + NI_1, \\ \phi(2) & = & \phi(5) = 2 + NI_1. \end{array}$$

It can easily be shown that ϕ is a NeutroRingHomomorphism with $Ker\phi = \{0, 3\} = NI_1$.

Example 3.23. Let $NR/NI_3 = \{NI_3, 1 + NI_3, 2 + NI_3, 3 + NI_3\}$ be the NeutroQuotientRing of Example 3.18 and let $\phi : NR \to NR/NI_3$ be a mapping defined by $\phi(x) = x + NI_3$ $\forall x \in NR$. Then

$$\begin{array}{rcl} \phi(0) & = & NI_3, \\ \phi(1) & = & 1 + NI_3, \\ \phi(2) & = & 2 + NI_3, \\ \phi(3) & = & 3 + NI_3. \end{array}$$

It can easily be shown that ϕ is a NeutroRingHomomorphism with $Ker\phi = 4\mathbb{Z} = NI_3$.

Proposition 3.24. *Let NR and NS be two NeutroRings of type-NR[8,9] and suppose that $\phi : NR \to NS$ is a NeutroRingHomomorphism. Then:*

(i) $\phi(e_{NR}) = e_{NS}$.

(ii) $Ker\phi$ *is a NeutroIdeal of* NR.

(iii) $Im\phi$ *is a NeutroSubring of* NS.

(iv) ϕ *is NeutroInjective if and only if* $Ker\phi = \{e_{NR}\}$.

Proof. The proof is the same as for the classical rings and so omitted. □

Proposition 3.25. *Let NI be a NeutroIdeal of the NeutroRing NR of type-NR[8,9]. The mapping $\psi : NR \to NR/NI$ defined by*

$$\psi(x) = x + NI \quad \forall x \in NR$$

is a NeutroRingEpimomorphism and the $Ker\psi = NI$.

Proof. The proof is the same as for the classical rings and so omitted. □

Proposition 3.26. [Fundamental Theorem of NeutroRingHomomorphisms]. *Let NR and NS be NeutroRings of type-NR[8,9] and let $\phi : NR \to NS$ be a NeutroRingHomomorpism with $K = Ker\phi$. Then the mapping $\psi : NR/K \to Im\phi$ defined by*

$$\psi(x + K) = \phi(x) \quad \forall x \in NR$$

is a NeutroRingIsomorphism.

Proof. The proof is the same as for the classical rings and so omitted. □

4 Conclusion

We have in this paper revisited the concept of NeutroRings introduced by Agboola in [5]. It was shown that there are 511 types of NeutroRings and 19171 types of AntiRings. In particular, we have studied finite and infinite NeutroRings of type-NR[8,9]. In the class of NeutroRings of type-NR[8,9], the left and right distributive axioms were taking to be either partially true or partially false for some elements; while all other classical laws and axioms were taking to be totally true for all the elements. Several examples and properties of NeutroRings of type-NR[8,9] were presented. NeutroSubrings, NeutroIdeals, NeutroQuotientRings and NeutroRingHomomorphisms of the NeutroRings of type-NR[8,9] were studied with several interesting examples and their basic properties were presented. It was shown that in the class of NeutroRings of type-NR[8,9], the fundamental theorem of homomorphisms of the classical rings holds.

5 Acknowledgment

The author is very grateful to Professor Florentin Smarandache for his valuable private discussions on the paper. The useful comments and suggestions of the anonymous reviewers are acknowledged.

References

[1] Agboola, A.A.A., Introduction to AntiGroups, Submitted for publication in International Journal of Neutrosophic Science (IJNS).

[2] Agboola, A.A.A., On Finite NeutroGroups of Type-NG[1,2,4], To appear in International Journal of Neutrosophic Science (IJNS).

[3] Agboola, A.A.A., Ibrahim, M.A. and Adeleke, E.O., Elementary Examination of NeutroAlgebras and AntiAlgebras viz-a-viz the Classical Number Systems, International Journal of Neutrosophic Science, vol. 4 (1), pp. 16-19, 2020. DOI:10.5281/zenodo.3752896.

[4] Agboola, A.A.A., Introduction to NeutroGroups, International Journal of Neutrosophic Science (IJNS), Vol. 6 (1), pp. 41-47, 2020. (DOI: 10.5281/zenodo.3840761).

[5] Agboola, A.A.A., Introduction to NeutroRings, International Journal of Neutrosophic Science (IJNS), Vol. 7 (2), pp. 62-73, 2020. (DOI:10.5281/zenodo.3877121).

[6] Gilbert, L. and Gilbert, J., Elements of Modern Algebra, Eighth Edition, Cengage Learning, USA, 2015.

[7] Rezaei, A. and Smarandache, F., On Neutro-BE-algebras and Anti-BE-algebras (revisited), International Journal of Neutrosophic Science (IJNS), Vol. 4 (1), pp. 08-15, 2020. (DOI: 10.5281/zenodo.3751862).

[8] Smarandache, F., NeutroAlgebra is a Generalization of Partial Algebra, International Journal of Neutrosophic Science, vol. 2 (1), pp. 08-17, 2020.

[9] Smarandache, F., Introduction to NeutroAlgebraic Structures and AntiAlgebraic Structures (revisited), Neutrosophic Sets and Systems (NSS), vol. 31, pp. 1-16, 2020. DOI: 10.5281/zenodo.3638232.

[10] Smarandache, F., Introduction to NeutroAlgebraic Structures, in Advances of Standard and Nonstandard Neutrosophic Theories, Pons Publishing House Brussels, Belgium, Ch. 6, pp. 240-265, 2019.

[11] Smarandache, F., Neutrosophy, Neutrosophic Probability, Set, and Logic, ProQuest Information & Learning, Ann Arbor, Michigan, USA, 105 p., 1998.
http://fs.unm.edu/eBook-Neutrosophic6.pdf (edition online).

The General Exponential form of a Neutrosophic Complex Number

Yaser Ahmad Alhasan

Deanship of the Preparatory Year, Prince Sattam bin Abdulaziz University, KSA; y.alhasan@psau.edu.sa ; yasermath85@hotmail.com

Abstract

In this paper, the general exponential form of a neutrosophic complex number is defined by virtue of the formula for indeterminacy in the angle $(\theta + \vartheta I)$, where $(\theta + \vartheta I)$ is the indeterminate angle between two indeterminate parts of the coordinate axes $(x - axis\ and\ y - axis)$, and the general trigonometric form of a neutrosophic complex number is defined. In addition, we also provide theorems with proofs for how to find the conjugate of neutrosophic complex numbers by using the general exponential form, division of neutrosophic complex numbers by the general exponential form, multiplying two neutrosophic complex numbers by the general exponential form, and the inverted neutrosophic complex number by the general exponential form.

Keywords: classical neutrosophic numbers, neutrosophic complex numbers, indeterminacy, conjugate, the general exponential form

1. Introduction

As an alternative to the existing logic, Smarandache proposed the neutrosophic logic for representing a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy and contradiction, where the concept of neutrosophy is a new branch of philosophy introduced by Smarandache [4][8]. Moreover, he presented the definition of the standard form of neutrosophic real numbers and conditions for the division of two neutrosophic real numbers, and he further defined the standard form of neutrosophic complex numbers, and found root index $n \geq 2$ of a neutrosophic real and complex number [3][5], studied the concept of the neutrosophic probability [4][6] and the neutrosophic statistics [5][7]. Then, Smarandache initiated the concept of preliminary calculus of the differential and integral calculus, where he first introduced the notion of neutrosophic mereo-limit, mereo-continuity, mereoderivative and mereo-integral [1][8]. Y. Alhasan presented the concept of neutrosophic complex numbers and its properties that include the conjugate of neutrosophic complex numbers, division of neutrosophic complex numbers, the inverted neutrosophic complex numbers and the absolute value of a neutrosophic complex numbers along with theories related to the conjugate of neutrosophic complex numbers, the product of a neutrosophic complex number by its conjugate equals the absolute value of numbers [2]. Madeleine Al-Taha presented some results on single valued neutrosophic (weak) polygroups [10]. Edalatpanah proposed a new direct algorithm to solve the neutrosophic linear programming where the variables and the right-hand side represented triangular neutrosophic numbers [11]. Chakraborty used pentagonal neutrosophic number in networking problems and shortest path problems [12][13]. Neutrophology logic provided brilliant mathematical theories as a generalization of fuzzy and crisp logic, such as the neutrosophic set theory, it followed the introduction of the neutrosophic concepts in Literature [14] and Literature [15]. Wadei Al-Omeri introduced the

DOI:10.5281/zenodo.4165054

Received: Jun 25, 2020 Accepted: Octobre 14, 2020

concept of neutrosophic crisp sets, investigated the properties of continuous, open and closed maps in neutrosophic crisp topological spaces [16].

Complex numbers are of great importance in our daily life, as they greatly help us in making mathematical operations, and they also provide a system for finding solutions to mathematical equations that have no solutions in the real number group, and one of the most prominent tools in the field of electrical engineering, calculating electric voltage and measuring electric current.

This paper aims to study the neutrosophic logic in the complex numbers by defining the generalized exponential form of a neutrosophic complex number. We find the conjugate, inverted of neutrosophic complex number by the general exponential form, division, and multiplying of neutrosophic complex numbers by the general exponential form. Moreover, we also provide some examples that reinforce a easy understanding of the current paper.

2. Preliminaries

2.1 Neutrosophic Real Numbers [5]

Suppose that w is a neutrosophic number, then it takes the following standard form: $w = a + bI$, where a , b are real coefficients, and I represents the indeterminacy, such that $0.\mathrm{I} = 0$ and $\mathrm{I}^n = \mathrm{I}$ for all positive integers n.

2.2 Neutrosophic Complex Numbers [5]

Suppose that z is a neutrosophic complex number, then it takes the following standard form: $z = a + cI + bi + diI$, where a , b, c, d are real coefficients, and I represents the indeterminacy, such that $i^2 = -1 \Rightarrow i = \sqrt{-1}$.

Note: we can say that any real number can be considered as a neutrosophic number.

For example: $2 = 2 + 0.I$, $or: 2 = 2 + 0.I + 0.i + 0.i.I$

2.3 Multiplying two neutrosophic complex numbers [3]

Let z_1, z_2 be two neutrosophic complex numbers, where

$\mathrm{z}_1 = \mathrm{a}_1 + \mathrm{c}_1\mathrm{I} + \mathrm{b}_1\mathrm{i} + \mathrm{d}_1\mathrm{iI}$, $\mathrm{z}_2 = \mathrm{a}_2 + \mathrm{c}_2\mathrm{I} + \mathrm{b}_2\mathrm{i} + \mathrm{d}_2\mathrm{iI}$

Then:

$$z_1 . z_2 = (a_1 + c_1 I + b_1 i + d_1 iI)(a_2 + c_2 I + b_2 i + d_2 iI)$$

$$= (a_1 a_2 - b_1 b_2) + (a_1 c_2 + a_2 c_1 + c_1 c_2 - b_1 d_2 - d_1 b_2 - d_1 d_2)I$$

$$+(a_1 b_2 + a_2 b_1)i + (a_1 d_2 + c_1 b_2 + c_1 d_2 + b_1 c_2 + a_2 d_1 + d_1 c_2)i.I$$

2.4 Division of neutrosophic real numbers [5]

Suppose that w_1, w_2 are two neutrosophic numbers, where

$$w_1 = a_1 + b_1 I \ , \qquad w_2 = a_2 + b_2 I$$

To find $(a_1 + b_1 I) \div (a_2 + b_2 I)$, we can write:

$$\frac{a_1 + b_1 I}{a_2 + b_2 I} \equiv x + yI$$

DOI:10.5281/zenodo.4165054

where x and y are real unknowns.

$$a_1 + b_1 I \equiv (a_2 + b_2 I)(x + yI)$$

$$a_1 + b_1 I \equiv a_2 x + (b_2 x + a_2 y + b_2 y)I$$

by identifying the coefficients, we get

$$a_1 = a_2 x$$

$$b_1 = b_2 x + (a_2 + b_{2)} y$$

We only obtain unique one solution, which is shown as follows:

$$\begin{vmatrix} a_2 & 0 \\ b_2 & a_2 + b_2 \end{vmatrix} \neq 0 \quad \Rightarrow \quad a_2(a_2 + b_2) \neq 0$$

Hence: $a_2 \neq 0 \quad and \;\; a_2 \neq -b_2$ are the conditions for the division of two neutrosophic real numbers to exist.

Then we have

$$\frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)}.I$$

2.5 The absolute value of a neutrosophic complex number[2]

Suppose that $z = a + cI + bi + d.iI$ is a neutrosophic complex number, the absolute value of a neutrosophic complex number defined by the following form:

$$|Z| = \sqrt{(a + cI)^2 + (b + dI)^2}$$

2.6 Conjugate of a neutrosophic complex number[2]

Suppose that z is a neutrosophic complex number, where $z = a + cI + bi + d.iI$. We denote the conjugate of a neutrosophic complex number by $\bar{z}$ and define it by the following form:

$$\bar{z} = a + cI - bi - d.iI$$

3. The general exponential form of a neutrosophic complex number

Theorem 3.1

The general exponential form of the neutrosophic complex number is given by the formula:

$$z = |z| e^{i(\theta + \vartheta I)} = r e^{i(\theta + \vartheta I)}$$

where $r = |z|$ is the absolute value of a neutrosophic complex number.

Proof:

We know that:

$$z = a + cI + bi + diI$$

DOI:10.5281/zenodo.4165054

$$z = \mathrm{r}\left(\frac{a+cI}{r} + \frac{b+dI}{r}\, i\right)$$

$$z = \mathrm{r}\left(\cos(\theta+\vartheta I) + sin\,(\theta+\vartheta I)\, i\right)$$

$$\cos(\theta+\vartheta I) = \frac{x}{r} = \frac{a\,+\,bI}{r} \quad , \quad sin\,(\theta+\vartheta I) = \frac{y}{r} = \frac{c\,+\,dI}{r}$$

Where $(\theta+\vartheta I)$ is the indeterminate angle between two indeterminate parts of the coordinate axes ($x - axis\ and\ y - axis$).

Hece we get:

$$z = re^{i(\theta+\vartheta I)}$$

3.1 The general Trigonometric form of a neutrosophic complex number

Definition 3.1:

The following formula:

$$z = \mathrm{r}\left(\cos(\theta+\vartheta I) + sin\,(\theta+\vartheta I)\, i\right)$$

is called the general Trigonometric form of a neutrosophic complex number.

3.2 Multiplying two neutrosophic complex numbers by the general exponential form

Theorem 3.2

Let $z_1 = r_1 e^{i(\theta_1+\vartheta_1 I)}$ and $z_2 = r_2 e^{i(\theta_2+\vartheta_2 I)}$

Then:

$$z_1 z_2 = r_1 r_2\, e^{i(\theta_1+\theta_2+(\vartheta_1+\vartheta_2)I)}$$

Proof:

$$z_1 z_2 = r_1 e^{i(\theta_1+\vartheta_1 I)} . r_2 e^{i(\theta_2+\vartheta_2 I)}$$

$$z_1 z_2 = r_1 r_2\big((cos(\theta_1+\vartheta_1 I) + i\sin(\theta_1+\vartheta_1 I))(cos(\theta_2+\vartheta_2 I) + i\sin(\theta_2+\vartheta_2 I))\big)$$

$$\begin{aligned} z_1 z_2 = r_1 r_2(\ &[cos(\theta_1+\vartheta_1 I)\, cos(\theta_2+\vartheta_2 I) - \sin(\theta_1+\vartheta_1 I)\sin(\theta_2+\vartheta_2 I)] \\ &+ i[\sin(\theta_1+\vartheta_1 I) cos(\theta_2+\vartheta_2 I) + cos(\theta_1+\vartheta_1 I)\sin(\theta_2+\vartheta_2 I)]) \end{aligned}$$

$$z_1 z_2 = r_1 r_2(\ \cos(\theta_1+\theta_2+(\vartheta_1+\vartheta_2)I) + i\sin(\theta_1+\theta_2+(\vartheta_1+\vartheta_2)I))$$

Hence:

$$z_1 z_2 = r_1 r_2\, e^{i(\theta_1+\theta_2+(\vartheta_1+\vartheta_2)I)}$$

Example 3.1

Let $z_1 = 2e^{i(\frac{\pi}{6}+\frac{\pi}{3}I)}$ and $z_2 = 4e^{i(\frac{\pi}{4}+\frac{\pi}{2}I)}$

DOI:10.5281/zenodo.4165054

Then:

$$z_1 z_2 = 8e^{i(\frac{5\pi}{12}+\frac{5\pi}{6}I)}$$

3.3 Conjugate of a neutrosophic complex number by the general exponential form

Theorem 3.3

Let $z = re^{i(\theta+\vartheta I)}$, then the conjugate of a neutrosophic complex number by the general exponential form is given by the formula:

$$\bar{z} = re^{-i(\theta+\vartheta I)}$$

Proof:

We know that:

$$\bar{z} = a + cI - bi - diI$$

$$\bar{z} = \mathrm{r}\left(\frac{a+cI}{r} - \frac{b+dI}{r}\, i\right)$$

$$\bar{z} = \mathrm{r}\,(\cos(\theta+\vartheta I) - \sin\,(\theta+\vartheta I)\, i)$$

$$\bar{z} = \mathrm{r}\,(\cos(-(\theta+\vartheta I)) + \sin\,(-(\theta+\vartheta I))\, i)$$

Hence we get:

$$\bar{z} = re^{-i(\theta+\vartheta I)}$$

3.4 Inverted Neutrosophic complex number by the general exponential form

Let $z = re^{i(\theta+\vartheta I)}$, then

$$\frac{1}{z} = \frac{1}{\mathrm{r}} e^{-i(\theta+\vartheta I)}$$

Proof:

$$\frac{1}{z} = \frac{1}{\mathrm{r}\,(\cos(\theta+\vartheta I) + \sin\,(\theta+\vartheta I)\, i)}$$

multiply the numerator and denominator by conjugate $(\cos(\theta+\vartheta I) - \sin\,(\theta+\vartheta I)\, i)$

then:

$$\frac{1}{z} = \frac{1}{\mathrm{r}} \frac{1}{(\cos(\theta+\vartheta I) + \sin\,(\theta+\vartheta I)\, i)} \cdot \frac{(\cos(\theta+\vartheta I) - \sin\,(\theta+\vartheta I)\, i)}{(\cos(\theta+\vartheta I) - \sin\,(\theta+\vartheta I)\, i)}$$

$$\frac{1}{z} = \frac{1}{\mathrm{r}} \frac{\cos(\theta+\vartheta I) - \sin\,(\theta+\vartheta I)\, i}{(\cos^2(\theta+\vartheta I) + \sin^2\,(\theta+\vartheta I)\,)} \qquad (*)$$

DOI:10.5281/zenodo.4165054

But:

$$\cos(\theta+\vartheta I)-sin\,(\theta+\vartheta I)i=e^{-i(\theta+\vartheta I)}$$

$$\cos^2(\theta+\vartheta I)+sin^2\,(\theta+\vartheta I)\;=1$$

By substitution in (*), we get:

$$\frac{1}{z}=\frac{1}{\mathrm{r}}e^{-i(\theta+\vartheta I)}$$

Special case:

When $r=1$, then:

$$\bar{z}=\frac{1}{z}$$

Example 3.2

Let $\mathrm{z}=2e^{i(\frac{\pi}{6}+\frac{\pi}{3}I)}$

Then:

$$\frac{1}{z}=\frac{1}{2}e^{-i(\frac{\pi}{6}+\frac{\pi}{3}I)}$$

Example 3.3

Let $\mathrm{z}=e^{i(\frac{\pi}{4}+\frac{\pi}{2}I)}$

Then:

$$\frac{1}{z}=e^{-i(\frac{\pi}{4}+\frac{\pi}{2}I)}$$

3.5 Division two neutrosophic complex numbers by the general exponential form

Theorem 3.4

Let $z_1=r_1e^{i(\theta_1+\vartheta_1 I)}$ *and* $z_2=r_2e^{i(\theta_2+\vartheta_2 I)}$

Then:

$$\frac{z_1}{z_2}=\frac{r_1}{r_2}e^{i(\theta_1-\theta_2+(\vartheta_1-\vartheta_2)I)}$$

Proof:

$$\frac{z_1}{z_2}=z_1.\frac{1}{z_2}$$

$$\frac{z_1}{z_2}=r_1e^{i(\theta_1+\vartheta_1 I)}.\frac{1}{r_2}e^{-i(\theta_2+\vartheta_2 I)}$$

DOI:10.5281/zenodo.4165054

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1+\vartheta_1 I)} . e^{-i(\theta_2+\vartheta_2 I)}$$

Hence we get:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1-\theta_2+(\vartheta_1-\vartheta_2)I)}$$

Example 3.4

Let $z_1 = 2e^{i(\frac{\pi}{6}+\frac{\pi}{3}I)}$ and $z_2 = 4e^{i(\frac{\pi}{4}+\frac{\pi}{2}I)}$

Then:

$$z_1 z_2 = \frac{1}{2} e^{-i(\frac{\pi}{12}+\frac{\pi}{6}I)}$$

5. Conclusions

In this paper, we define the general exponential form of a neutrosophic complex number, and arithmetic perations on it, multiplication, division, conjugate and of a neutrosophic complex number by the general exponential form, and find inverted neutrosophic complex numbers by the general exponential form. This research is considered as one of the important researches in neutrosophic complex numbers.

References

[1] F.Smarandache, "Introduction to Neutrosophic Measure, Neutrosophic Integral, and Neutrosophic Probability", Sitech-Education Publisher, Craiova – Columbus, 2013.

[2] Y.Alhasan, "Concepts of Neutrosophic Complex Numbers", International Journal of Neutrosophic Science, Volume 8 , Issue 1, pp: 9-18, 2020.

[3] F.Smarandache "Finite Neutrosophic Complex Numbers, by W. B. Vasantha Kandasamy", Zip Publisher, Columbus, Ohio, USA, PP:1-16, 2011.

[4] F.Smarandache, "Neutrosophy. / Neutrosophic Probability, Set, and Logic", American Research Press, Rehoboth, USA, 1998.

[5] F.Smarandache, "Introduction to Neutrosophic statistics", Sitech-Education Publisher, PP:34-44, 2014.

[6] F.Smarandache, "A Unifying Field in Logics: Neutrosophic Logic, Preface by Charles Le, American Research Press, Rehoboth, 1999, 2000. Second edition of the Proceedings of the First International Conference on Neutrosophy, Neutrosophic Logic, Neutrosophic Set, Neutrosophic Probability and Statistics", University of New Mexico, Gallup, 2001.

[7] F.Smarandache, "Proceedings of the First International Conference on Neutrosophy", Neutrosophic Set, Neutrosophic Probability and Statistics, University of New Mexico, 2001.

[8] F.Smarandache, "Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics" University of New Mexico, Gallup, NM 87301, USA 2002.

[9] F.Smarandache, "Neutrosophic Precalculus and Neutrosophic Calculus", book, 2015.

[10] Madeleine Al- Tahan, "Some Results on Single Valued Neutrosophic (Weak) Polygroups", International Journal of Neutrosophic Science, Volume 2 , Issue 1, PP: 38-46 , 2020.

[11] S. A. Edalatpanah, "A Direct Model for Triangular Neutrosophic Linear Programming", International Journal of Neutrosophic Science, Volume 1 , Issue 1, PP: 19-28 , 2020

[12] A. Chakraborty, "A New Score Function of Pentagonal Neutrosophic Number and its Application in Networking Problem", International Journal of Neutrosophic Science, Volume 1 , Issue 1, PP: 40-51 , 2020

DOI:10.5281/zenodo.4165054

[13] A. Chakraborty, "Application of Pentagonal Neutrosophic Number in Shortest Path Problem", International Journal of Neutrosophic Science, Volume 3 , Issue 1, PP: 21-28 , 2020
[14] A.A. Salama and S.A. Alblowi, "Neutrosophic set and neutrosophic topological space". ISORJ, Mathematics,Volume 3, Issue 4, PP: 31-35, 2012
[15] A. A. Salama and F. Smarandache. Filters via, "Neutrosophic Crisp Sets". Neutrosophic Sets and Systems, Volume 1, PP: 34-38, 2013
[16] W. Al-Omeri, "Neutrosophic crisp sets via neutrosophic crisp topological spaces". Neutrosophic Sets and Systems, volume 13, PP: 96–104, 2016

Pythagorean Neutrosophic Fuzzy Graphs

D. Ajay [1,*], P. Chellamani [2]

[1] Department of Mathematics, Sacred Heart College (Autonomous), Tirupattur – 635 601, Tirupattur District, Tamil Nadu, INDIA; dajaypravin@gmail.com

[2] Department of Mathematics, Sacred Heart College (Autonomous), Tirupattur – 635 601, Tirupattur District, Tamil Nadu, INDIA; joshmani238@gmail.com

* Correspondence: dajaypravin@gmail.com

Abstract

In this paper, we present the new idea of pythagorean neutrosophic fuzzy graphs (PNFG). Pythagorean neutrosophic set [PNS] is a generalization of neutrosophic set with dependent neutrosophic components and Pythagorean fuzzy set with condition $0 \leq \mu_A(x)^2 + \beta_A(x)^2 + \sigma_A(x)^2 \leq 2$. The main aim of this article is to apply pythagorean neutrosophic set to fuzzy graphs. Thus we extend some of the basic properties for this PNFG along with few examples.

Keywords: Pythagorean neutrosophic set, Pythagorean neutrosophic graphs, Neutrosophic set.

1.Introduction

L.A. Zadeh originated the idea of fuzzy set theory in 1965 [1]. Atanassov [2] presented a new set intuitionistic set as an extension of fuzzy set which defines element with the membership and non-membership grade with restriction that totality of both must not exceed 1. The new perception of pythagorean fuzzy sets was published by Yager [3] in which the sets has constraint that their square sum is ≤ 1. From the definition of pythagorean sets, pythagorean fuzzy graphs have been developed and is applied in many fields [12].

Neutrosophic set, a combination of fuzzy and intuitionistic sets was introduced by Smarandache [4] in 1995. Many researchers have developed this results and applied in many relevant fields as decision making and so on [13-16]. A neutrosophic set has truth, falsity and indeterminacy membership for each element. In particular, the neutrosophic set has been applied to fuzzy graph theory and many new concepts of neutrosophic fuzzy graph has been developed [8-11, 17-18]. Smarandache introduced the degree of dependence among components of fuzzy set and in neutrosophic set. Then the researchers studied about neutrosophic and developed many new concepts in neutrosophic sets and in neutrosophic graphs magic labellling was applied [6-7]. In neutrosophic set, out of three dependency cases we choose one special case in which indeterminacy is independent, truth and falsity are completely dependent with $0 \leq \mu_A(x)^2 + \beta_A(x)^2 + \sigma_A(x)^2 \leq 2$ is termed as pythagorean neutrosophic set. In this research, we put on the pythagorean neutrosophic sets to fuzzy graphs and also we study some types of different notions of pythagorean neutrosophic graph.

DOI: 10.5281/zenodo.4172124

Received: July 18, 2020 Accepted: Octobre 20, 2020

2. Preliminaries

An intuitionistic fuzzy set I on universe Y is given by $I = \{ < s,\ \mu_I(s),\ \sigma_I(s) >: s \in Y \}$ where μ_I, σ_I from Y to $[0,1]$ such that $0 \leq \mu_I(s) + \sigma_I(s) \leq 1$ for any $s \in Y$. $\mu_I(s)$ and $\sigma_I(s)$ are the membership and non-membership grade of s correspondingly.

A neutrosophic set N in Y is given by $N = \{(\mathfrak{k},\ \mu_N(\mathfrak{k}),\ \eta_N(\mathfrak{k}),\ \gamma_N(\mathfrak{k})) \mid \mathfrak{k} \in Y)\}$, where $\mu_N(\mathfrak{k}),\ \eta_N(\mathfrak{k}),\ \gamma_N(\mathfrak{k}) \in [0,1]$ denote truth, indeterminacy and false membership of $\mathfrak{k}$ of N and $\mu_N(\mathfrak{k}),\ \eta_N(\mathfrak{k}),\ \gamma_N(\mathfrak{k})$ follow the condition that $0 \leq \mu_N(\mathfrak{k}) + \eta_N(\mathfrak{k}) + \gamma_N(\mathfrak{k}) \leq 3$.

A neutrosophic fuzzy graph is $G = (V, E)$ where $V = \{v_1, v_2, \dots v_n\}$ such that μ_1, β_1 and σ_1 from V to $[0,1]$ represent degree of truth-membership, indeterminacy-membership and falsity-membership of c_i in V correspondingly, and $0 \leq \mu_1(c_i) + \beta_1(c_i) + \sigma_1(c_i) \leq 3 \quad \forall\, c_i \in V\ (i = 1, \dots n)$, $E \subseteq V \times V$ with μ_2, β_2 and σ_2 from V x V to $[0,1]$ such that

$$\mu_2(c_i c_j) \leq \mu_1(c_i) \wedge \mu_1(c_j)$$

$$\beta_2(c_i c_j) \leq \beta_1(c_i) \wedge \beta_1(c_j)$$

$$\sigma_2(c_i c_j) \leq \sigma_1(c_i) \vee \sigma_1(c_j)$$

$$0 \leq \mu_2(c_i c_j) + \beta_2(c_i c_j) + \sigma_2(c_i c_j) \leq 3 \text{ for every } (c_i c_j) \in E\ (i, j = 1,2, \dots n).$$

Pythagorean fuzzy set (PFS) set P of Y is $P = \{ < \mathfrak{r},\ \mu_P(\mathfrak{r}),\ \sigma_P(\mathfrak{r}) >: \mathfrak{r} \in Y\}$ where $\mu_P(\mathfrak{r})$ and $\sigma_P(\mathfrak{r})$from Y to $[0,1]$ represents degree of membership and non-membership of $\mathfrak{r}$ in P correspondingly. $\forall\, \mathfrak{r} \in Y$, the following condition should be satisfied $0 \leq \mu_P^2(\mathfrak{r}) + \sigma_P^2(\mathfrak{r}) \leq 1$.

A Pythagorean fuzzy graph (PFG) is a duo $G = (V, E)$ with μ_1 and σ_1 from V to $[0,1]$ signifying membership, non-membership functions of V and $0 \leq \mu_1^2(v) + \sigma_1^2(v) \leq 1$ for $v \in V$ such that

$$\mu_2(uv) \leq \mu_1(u) \wedge \mu_1(v),$$

$$\sigma_2(uv) \leq \sigma_1(u) \vee \sigma_1(v).$$

where μ_2, σ_2 from V x V to $[0,1]$ represent the membership, non-membership functions of E, with $0 \leq \mu_2^2(uv) + \sigma_2^2(uv) \leq 1$ for all $uv \in E$.

A Pythagorean Neutrosophic set with truth, falsity as dependent neutrosophic components [PNS] on non-empty universe Y is $D = \{(d,\ \mu_D(d), \beta_D(d), \sigma_D(d)): d \in Y\}$, where $\mu_D(d),\ \beta_D(d),\ \sigma_D(d) \in [0, 1]$, $0 \leq (\mu_D(d))^2 + (\beta_D(d))^2 + (\sigma_D(d))^2 \leq 2, \quad \forall\, d \in Y$. $\mu_D(d),\ \beta_D(d),\ \sigma_D(d)$ are the degrees of membership, indeterminacy and non-membership respectively. Here $\mu_D(d)$ and $\sigma_D(d)$ are dependent and $\beta_D(d)$ is independent [5].

3. Pythagorean Neutrosophic Fuzzy Graphs

Definition 3.1: Pythagorean Neutrosophic Fuzzy Graph (PNFG) is $G = (V, E)$, where $V = \{v_1, v_2, \dots v_n\}$ such that μ_1, β_1 and $\sigma_1$from V to $[0,1]$ with $0 \leq \mu_1(v_i)^2 + \beta_1(v_i)^2 + \sigma_1(v_i)^2 \leq 2\ \forall\, v_i$ in V signifies membership, indeterminacy and non-membership functions correspondingly and $E \subseteq V$ x V where μ_2, β_2 σ_2 from V x V to $[0,1]$ such that

$$\mu_2(v_i v_j) \leq \mu_1(v_i) \wedge \mu_1(v_j)$$

$$\beta_2(v_iv_j) \leq \beta_1(v_i) \wedge \beta_1(v_j)$$

$$\sigma_2(v_iv_j) \leq \sigma_1(v_i) \vee \sigma_1(v_j)$$

With $0 \leq \left((\mu_2(v_iv_j))\right)^2 + \left(\beta_2(v_i\, v_j)\right)^2 + \left(\sigma_2(v_iv_j)\right)^2 \leq 2\ \forall\ (v_iv_j) \in E$.

The example given in figure 1 is a PNFG

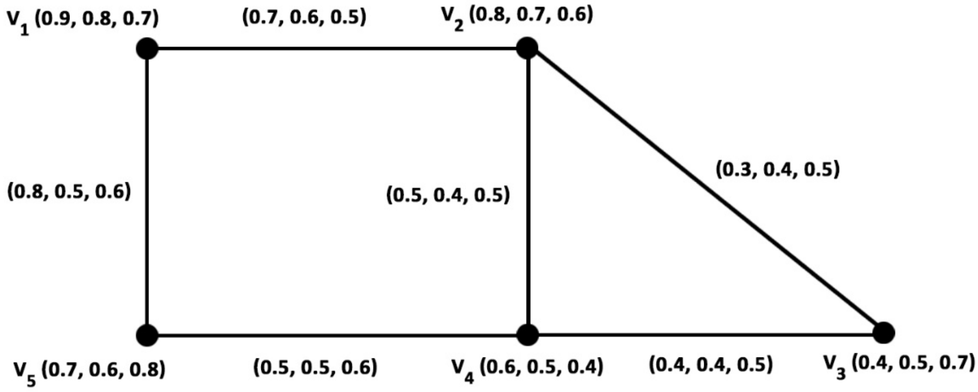


Figure 1. Pythagorean Neutrosophic Fuzzy Graph

Definition 3.2: A PNFG $G = (V, E)$ is termed as complete PNFG (CPNFG) if $\mu_2(v_iv_j) = \mu_1(v_i) \wedge \mu_1(v_j)$, $\beta_2(v_iv_j) = \beta_1(v_i) \wedge \beta_1(v_j)$, $\sigma_2(v_iv_j) = \sigma_1(v_i) \vee \sigma_1(v_j)$ for every $v_i, v_j \in V$.

Definition 3.3: A PNFG $G = (V, E)$ is named as strong PNFG if

$$\mu_2(v_iv_j) = \min\left(\mu_1(v_i), \mu_1(v_j)\right)$$

$$\beta_2(v_iv_j) = \min\left(\beta_1(v_i), \beta_1(v_j)\right)$$

$$\sigma_2(v_iv_j) = \max\left(\sigma_1(v_i), \sigma_1(v_j)\right)\ \forall\ (v_iv_j) \in E.$$

Definition 3.4: Let $G = (V, E)$ with μ, β, σ as the membership (MD), indeterminacy (ID) and non-membership (NMD) degree be a PNFG. Then a PNFG $H = (V', E')$ with $V' \subseteq V$ and $E' \subseteq E$, μ', β' and σ' as the MD, NMD, and ID is called Pythagorean Neutrosophic fuzzy subgraph (PNFSG) if $\mu'(\mathfrak{d}) \leq \mu(\mathfrak{d})$, $\beta'(\mathfrak{d}) \leq \beta(\mathfrak{d})$, $\sigma'(\mathfrak{d}) \geq \sigma(\mathfrak{d})$ for $\mathfrak{d} \in V$.

Definition 3.5: Let $G' = (V', E')$, $G'' = (V'', E'')$ be PNFG with (μ', β', σ') and $(\mu'', \beta'', \sigma'')$ as their MD, ID and NMD correspondingly. Then the intersection of G' and G'', $G = (V, E)$ is a PNFG where $V = V' \cap V''$, $E = E' \cap E''$ and the MD, ID and NMD of V and E of for all $a, b, r \in V$ such that

(i) $\mu_1(r) = \begin{cases} \mu_1'(r) & if\ r \text{ is in } V' \text{ and not in } V'' \\ \mu_1''(r) & if\ r \text{ is in } V'' \text{and not in } V' \\ \mu_1'(r) \wedge \mu_1''(r) & if\ r \text{ is in both } V' \text{ and } V'' \end{cases}$

$$\beta_1(r) = \begin{cases} \beta_1'(r) & if\ r \text{ is in } V' \text{ and not in } V'' \\ \beta_1''(r) & if\ r \text{ is in } V'' \text{and not in } V' \\ \beta_1'(r) \wedge \beta_1''(r) & if\ r \text{ is in both } V' \text{ and } V'' \end{cases}$$

$$\sigma_1(r) = \begin{cases} \sigma_1'(r) & if\ r \text{ is in } V' \text{ and not in } V'' \\ \sigma_1''(r) & if\ r \text{ is in } V'' \text{and not in } V' \\ \sigma_1'(r) \vee \sigma_1''(r) & if\ r \text{ is in both } V' \text{ and } V'' \end{cases}$$

(i) $$\mu_2(ab) = \begin{cases} \mu_2'(ab) & if\ ab \text{ is in } E' \text{ and not in } E'' \\ \mu_2''(ab) & if\ ab \text{ is in } E'' \text{ and not in } E' \\ \mu_2'(ab) \wedge \mu_2''(ab) & if\ ab \text{ is in both } E' \text{ and } E'' \end{cases}$$

$$\beta_2(ab) = \begin{cases} \beta_2'(ab) & if\ ab \text{ is in } E' \text{ and not in } E'' \\ \beta_2''(ab) & if\ ab \text{ is in } E'' \text{ and not in } E' \\ \beta_2'(ab) \wedge \beta_2''(ab) & if\ ab \text{ is in both } E' \text{ and } E'' \end{cases}$$

$$\sigma_2(ab) = \begin{cases} \sigma_2'(ab) & if\ ab \text{ is in } E' \text{ and not in } E'' \\ \sigma_2''(ab) & if\ ab \text{ is in } E'' \text{ and not in } E' \\ \sigma_2'(ab) \vee \sigma_2''(ab) & if\ ab \text{ is in both } E' \text{ and } E'' \end{cases}$$

Definition 3.6: Let $G' = (V', E')$, $G'' = (V'', E'')$ be PNFG with $(\mu_1', \beta_1', \sigma_1')$, $(\mu_1'', \beta_1'', \sigma_1'')$ and $(\mu_2', \beta_2', \sigma_2')$, $(\mu_2'', \beta_2'', \sigma_2'')$ as the MD, ID and NMD of the vertices and edges correspondingly. Then the union of G' & G'', $G = (V, E)$ is a PNFG where $V = V' \cup V''$, $E = E' \cup E''$ and the MD, ID and NMD of vertices (V), edges (E) of G for all $g, h \in V$ such that

(i) $$\mu_1(g) = \begin{cases} \mu_1'(g) & if\ g \text{ is in } V' \text{ and not in } V'' \\ \mu_1''(g) & if\ g \text{ is in } V'' \text{and not in } V' \\ \mu_1'(g) \vee \mu_1''(g) & if\ g \text{ is in } V' \text{ or } V'' \end{cases}$$

$$\beta_1(g) = \begin{cases} \beta_1'(g) & if\ g \text{ is in } V' \text{ and not in } V'' \\ \beta_1''(g) & if\ g \text{ is in } V'' \text{and not in } V' \\ \beta_1'(g) \vee \beta_1''(g) & if\ g \text{ is in } V' \text{ or } V'' \end{cases}$$

$$\sigma_1(g) = \begin{cases} \sigma_1'(g) & if\ g \text{ is in } V' \text{ and not in } V'' \\ \sigma_1''(g) & if\ g \text{ is in } V'' \text{and not in } V' \\ \sigma_1'(g) \wedge \sigma_1''(g) & if\ g \text{ is in } V' \text{ or } V'' \end{cases}$$

(ii) $$\mu_2(gh) = \begin{cases} \mu_2'(gh) & if\ gh \text{ is in } E' \text{ and not in } E'' \\ \mu_2''(gh) & if\ gh \text{ is in } E'' \text{ and not in } E' \\ \mu_2'(gh) \vee \mu_2''(gh) & if\ gh \text{ is in } E' \text{ or } E'' \end{cases}$$

$$\beta_2(gh) = \begin{cases} \beta_2'(gh) & if\ gh \text{ is in } E' \text{ and not in } E'' \\ \beta_2''(gh) & if\ gh \text{ is in } E'' \text{ and not in } E' \\ \beta_2'(gh) \vee \beta_2''(gh) & if\ gh \text{ is in } E' \text{ or } E'' \end{cases}$$

$$\sigma_2(gh) = \begin{cases} \sigma_2'(gh) & if\ gh \text{ is in } E' \text{ and not in } E'' \\ \sigma_2''(gh) & if\ gh \text{ is in } E'' \text{ and not in } E' \\ \sigma_2'(gh) \wedge \sigma_2''(gh) & if\ gh \text{ is in } E' \text{ or } E'' \end{cases}$$

Note: The following notion is followed in this paper, If $G = (V, E, \rho, \gamma)$ where $\rho = (\mu_1, \beta_1, \sigma_1)$ and $\gamma = (\mu_2, \beta_2, \sigma_2)$ represents the MD, ID and NMD of the vertices and edges of PNFG respectively.

Definition 3.7: A Pythagorean Neutrosophic path $P(PNP)$ in PNG $G = (V, E, \rho, \gamma)$ is a arrangement of different vertices $v_0, v_1, \ldots v_n$ (leaving v_0, v_1) such that $\gamma(v_{i-1}, v_i) > 0$, $i = 1\ to\ n$ (where n is the length of the PNP).

The consecutive pair of the PNP are called the edges.

Definition 3.8: The diameter of a, b in V is the length of the longest PNP joining a and b and denoted as $diam\ (a, b)$.

The strength of PNP P is is represented by $d(P)$ or $S(P)$ and defined as

$$\bigwedge_{k=1}^{n} \gamma(v_{k-1}, v_k) = \left(\bigwedge_{k=1}^{n} \mu_2(v_{k-1}, v_k), \bigwedge_{k=1}^{n} \beta_2(v_{k-1}, v_k), \bigvee_{k=1}^{n} \sigma_2(v_{k-1}, v_k)\right) \quad \text{where } v_k \in V \ \ (k = 1,2, \dots, n)$$

Definition 3.9: The pythagorean neutrosophic strength of connectedness of vertices a and b of PNFG is defined as the maximum of the strength of all PNP's among a and b and represented by $PNCONN_G(x, y)$.

$PNCONN_G(x, y) = \max\ (S(P))$ where P is a $x - y$ PNP in G.

If $n \geq 3$ and $V_0 = V_n$ then PNP P is called a Pythagorean Neutrosophic Cycle (PNC).

Example 3.10: Consider the following PNFG

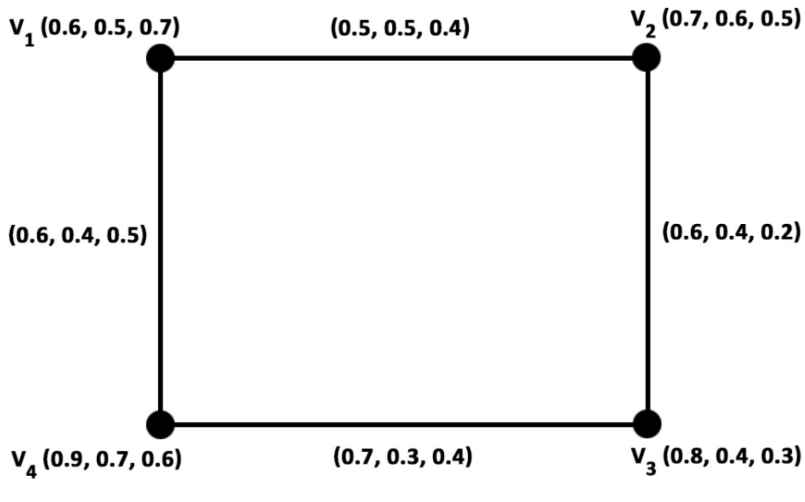


Figure 2. Pythagorean Neutrosophic strength of connectedness of PNFG

$$PNCONN_G(v_1, v_2) = \big(max(0.5,0.6), max(0.5,0.3), min(0.4,0.5)\big)$$

$$= (0.6, 0.5, 0.4)$$

$$PNCONN_G(v_1, v_3) = \big(max(0.5,0.6), max(0.4,0.3), min(0.4,0.5)\big)$$

$$= (0.6, 0.4, 0.4)$$

$$PNCONN_G(v_1, v_4) = \big(max(0.6,0.5), max(0.4,0.3), min(0.5,0.4)\big)$$

$$= (0.6, 0.4, 0.4)$$

Definition 3.11: Let $G = (V, E, \rho, \gamma)$ be a PNFG and *x, y* be two different vertices, let G' be a PNFSG of G attained by removing the edge *xy*. *xy* is a Pythagorean neutrosophic fuzzy bridge (PNFB) in G if $PNCONN_{G'}(a, b) < PNCONN_G(a, b)$ for some *a,b*. The removal of the edge *xy* decreases the strength of connectedness among some duo of vertices in G. Thus, *xy* is a PNFB if and only if there exists vertices a, b such that *xy* is an edge of each strongest path from a to b.

Theorem 3.12: Let $G = (V, E, \rho, \gamma)$ be a PNFG. Then the subsequent statements are equivalent.

1. *xy* is a PNFB
2. $PNCONN_{G'}(x, y) < \gamma(xy)$
3. *xy* is not the weakest edge of any Pythagorean neutrosophic cycle (PNC)

Proof:

2 ⇒1 If *xy* is not a PNFB, then $PCONN_{G'}(x, y) = PCONN_G(x, y) \geq \gamma(xy)$.

1 ⇒ 3 If *xy* is the weakest edge of a PNC, then any PNP *P* including the edge *xy* can be converted into a PNP P' not involving *xy* but at least as strong as *P*, by using the rest of the PNC as a PNP from *x* to *y*. Thus, *xy* cannot be a PNFB.

3 ⇒ 2 If $PCONN_{G'}(x, y) < \gamma(xy)$, there is a PNP from x to *y* not including *xy* with strength $\geq \gamma(xy)$, and this PNP together with *xy* forms a PNC of *G* in which *xy* is a weakest edge.

Definition 3.13: Let *w* be any vertex and let $G' = (V', E', \rho', \gamma')$be a PNFSG of $G = (V, E, \rho, \gamma)$ attained by removing the vertex *w*. That is, $G' = (V', E', \rho', \gamma')$ is the PNFSG of G such that $\rho'(w) = 0, \rho' = \rho$ for all other vertices, $\gamma'(wz) = 0$ for all vertices *z*, and $\gamma' = \gamma$ for all other edges. Thus we call *w* a Pythagorean neutrosphic fuzzy cutvertex in G if $PNCONN_{G'}(u, v) < PNCONN_G(u, v)$ for some *u, v* in V such that $u \neq w \neq v$.

5. Conclusions

Herein, we have defined a new concept pythagorean neutrosophic graphs by applying pythagorean neutrosophic set to fuzzy graph. Also have defined some of its basic definitions and properties of the pythagorean neutrosophic graphs. In future, we would extend this by introducing more definitions and apply labelling, coloring to PNFG.

Funding: "This research received no external funding"

Conflicts of Interest: "The authors declare no conflict of interest."

References

[1] Zadeh, L.A., "Fuzzy Sets", Inform. and Control, Vol 8, pp.338- 353, 1965.
[2] Atanassov, K., "Intuitionistic Fuzzy Sets", Fuzzy Sets and Systems, Vol 20, pp.87-96, 1986.
[3] Yager, R.R., "Pythagorean Fuzzy Subsets", In:Proc Joint IFSA World Congress and NAFIPS Annual Meeting, Edmonton, Canada, pp.57-61, 2013.
[4] Smarandache, F., "A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic Set, Neutrosophic Probability", American Research Press: Rehoboth, DE, USA, pp.1-141, 1999.
[5] Jansi, R., Mohana, R.K., & Smarandache, F., "Correlation Measure for Pythagorean Neutrosophic Fuzzy Sets with T and F as Dependent Neutrosophic Components", Neutrosophic Sets and Systems, Vol 30(1), pp.202-212, 2019.
[6] Ajay, D., Charisma, J. J., & Chellamani, P., "Fuzzy Magic and Bi-magic Labelling of Intuitionistic Path Graph", International Journal of Recent Technology and Engineering, Vol 8(4), pp.11508-11512, 2019.

[7] Ajay, D., & Chellamani, P., "Fuzzy Magic Labelling of Neutrosophic Path and Star graph", Advances in Mathematics: Scientific Journal Vol 9(8), pp.6059-6070, 2020.
[8] Hussain, S.S., Hussain, R.J., Jun, Y.B., & Smarandache, F., "Neutrosophic bipolar vague set and its application to neutrosophic bipolar vague graphs", Neutrosophic Sets and Systems, Vol 28, pp.69-86, 2019.
[9] Hussain, S.S., Hussain, R.J., & Muhiuddin, G., "Neutrosophic Vague Line Graphs", Neutrosophic Sets and Systems, Vol 36, pp.121-130. 2020.
[10] Hussain, S.S., Hussain, R.J., & Smarandache, F., "On neutrosophic vague graphs", Neutrosophic Sets and Systems, Vol 28, pp.245-258, 2019.
[11] Dhavaseelan, R., Jafari, S., Farahani, M.R., & Broumi, S., "On single-valued co-neutrosophic graphs", Neutrosophic Sets and Systems: An International Book Series in Information Science and Engineering, vol. 22/2018, 22. 2018.
[12] Akram, M., Khan, A., & Borumand Saeid, A., "Complex Pythagorean Dombi fuzzy operators using aggregation operators and their decision□making", Expert Systems, pp.1-28, 2020.
[13] Ajay, D., Broumi, S., & Aldring, J., "An MCDM Method under Neutrosophic Cubic Fuzzy Sets with Geometric Bonferroni Mean Operator", Neutrosophic Sets and Systems, Vol 32(1), pp.187-202, 2020.
[14] Ajay, D., Manivel, M., & Aldring, J., "Neutrosophic Fuzzy SAW Method and It's Application", Vol 11, pp.881-887, 2019.
[15] Ajay, D., Aldring, J., Abirami, S., & Jeni Seles Martina, D., "A SVTrN-number approach of muliti-objective optimization on the basis of simple ration analysis based on MCMD method", International Journal of Neutrosophic Science, Vol 5(1), 16-28, 2020.
[16] Ajay, D., & Aldring, J., "A Decision Making Technique Based on Similarity Measure and Entropy of Bipolar Neutrosophic Sets", The International journal of analytical and experimental modal analysis, Vol 11(9), pp.520-529, 2019.
[17] Mullai, M., Broumi, S., & Jeyabalan, R., "Homomorphism and Isomorphism in strong neutrosophic graphs", International Journal of Neutrosophic Science, Vol 0, pp.08-20, 2020.
[18] Mullai, M., & Broumi, S., "Dominating energy in neutrosophic graphs", International Journal of Neutrosophic Science, Vol 5(1), pp.38-58, 2020.